Lesson 1.4 — Teacher Reference Notes

Angle Relationships and the First Proof

Unit 1: Foundations of Geometry • Day 5 of unit • can be done in one 90-minute block

At a Glance	**Pacing:** Angle naming activator (8 min) • Task 1: Interior/exterior (15 min) • Task 2: Two intersecting lines (25 min) • Consolidation + notes + first proof (35 min) • CYU begins (7 min) **Materials:** Boards and markers • Protractors (one per group) • Angle naming activator images (Math Medic link below) **Nav instrument rows active:** Rows 1–4 spiral • Row 5 (Angle Relationships) activates today **Note:** This lesson was taught over two days with daily quizzes. With the quiz constraint removed, one 90-minute block is sufficient. Do not let the naming activator crowd out the proof work.

OBJECTIVES

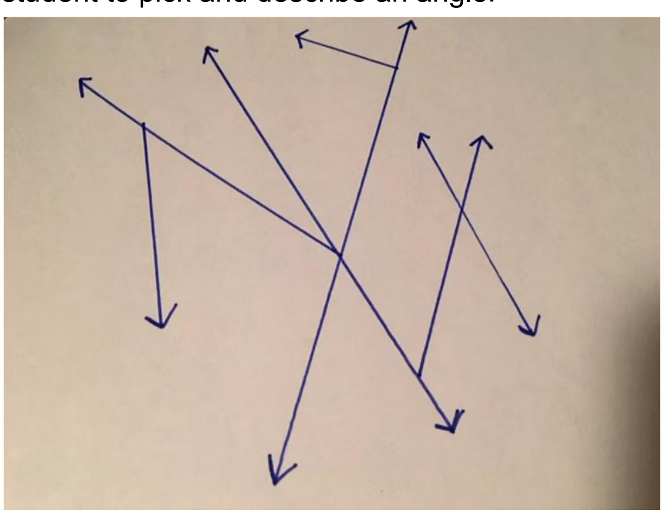
By the end of this lesson students:

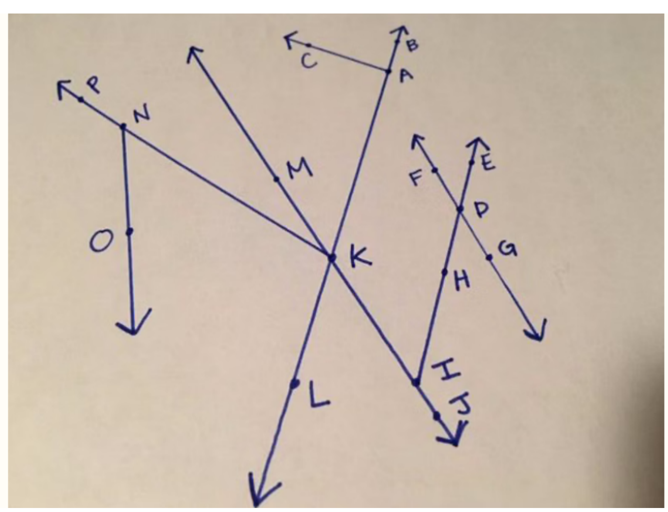
- Can name angles unambiguously using three-point notation.
- Can identify and define linear pairs, vertical angles, supplementary, complementary, and adjacent angles.
- Can use angle relationships to set up and solve algebraic equations.
- Can construct a convincing argument that vertical angles must be congruent.

ACTIVATOR — WHEN DOES ANGLE A FAIL? (ADAPTED FROM MATH MEDIC)

Source: portal.mathmedic.com/lesson-plans/course/Geometry/unit/2/day/3

Show the first diagram from below. Ask a student to pick an angle and try to describe to us which one it is. You may choose to do this with a few students but eventually show the second image and ask a new student to pick and describe an angle.





Make sure to ask students if you just said angle K, if they would know which angle you were talking about/

> **After the ambiguity lands**
>
> *Introduce three-point notation: the vertex in the middle, one point on each ray. Angle BAC and angle CAD are now distinct. Practice: have students write three-point notation for three specific angles in the diagram before moving to Task 1.*

TASKS

Task 1 — Interior and Exterior Angles (≈15 min)

Each group draws one angle and labels it. Then say:

> **Prompt**
>
> **I asked you to draw one angle. It appears to me you drew two.**

Let groups explore until they identify the interior and exterior angle. Measure both using a protractor. To measure the exterior, extend one ray.

Conjecture to reach: what do the interior and exterior angles together sum to? Always?

Name the move explicitly: extending a ray to reveal the exterior angle is itself a mathematical move — adding to a diagram to see what is not explicitly drawn. This habit is essential in proof work later.

Task 2 — Two Intersecting Lines (≈25 min)

> **Prompt**
>
> **Draw two lines that cross at any non-right angle. Use your mathematical thinking. Play, wonder, conjecture, test, generalize, convince. Come up with as many conjectures as you can.**

Make sure groups use protractors to test their conjectures. Practice the skill and practice looking for evidence rather than assuming.

> **Target conjectures**
>
> - Linear pairs sum to 180°
> - Vertical angles are congruent

⚠️ **Once both are conjectured — push toward the argument**

Ask: how do we know vertical angle congruence must always be true? Not just for these specific angles — always. Measuring confirms it for the angles in front of us. But we want something stronger. Can you construct an argument from what we already know?

The argument is accessible: both vertical angles form a linear pair with the same angle. Linear pairs sum to 180°. So both vertical angles equal 180° minus the same quantity. Therefore they must be equal. Let groups develop this at the boards before consolidating.

CONSOLIDATION — DEFINITIONS AND THE FIRST PROOF

Define: linear pair, vertical angles, supplementary, complementary, adjacent. Connect each to the board work — students should recognize the examples from their own diagrams.

ANGLE NAMING

Angles are named using three points: one point on each ray with the vertex in the middle. Write the vertex letter in the center.

Symbol: $\angle ABC$ — vertex at B, one point on each ray (A and C)

Single-letter naming (e.g., $\angle B$) is only acceptable when exactly one angle is formed at that vertex. As soon as two or more angles share a vertex, three-point notation is required.

ANGLE RELATIONSHIP DEFINITIONS

Linear Pair

Two adjacent angles whose non-common sides form a straight line (a straight angle of 180°). A linear pair is always supplementary.

If $\angle ABC$ and $\angle CBD$ form a linear pair, then $m\angle ABC + m\angle CBD = 180°$

Key fact used in the vertical angles proof: if two angles form a linear pair, they are supplementary — their measures sum to 180°.

Vertical Angles

Two non-adjacent angles formed by two intersecting lines. Vertical angles share a vertex but no sides; they sit across the intersection from each other.

$\angle ABC \cong \angle DBE$ when lines AD and CE intersect at B

Theorem (proven this lesson): vertical angles are always congruent.

Supplementary Angles

Two angles whose measures sum to 180°. They do not need to be adjacent.

$m\angle A + m\angle B = 180°$

Note: a linear pair is a special case of supplementary angles that ARE adjacent and form a straight line.

Complementary Angles

Two angles whose measures sum to 90°. They do not need to be adjacent.

$m\angle A + m\angle B = 90°$

Adjacent Angles

Two angles that share a common vertex and a common side, but have no interior points in common (one does not overlap the other).

Adjacent angles are not necessarily supplementary or complementary — they simply share a vertex and a side.

Then turn to the vertical angles argument. Brief gallery walk: ask students to look at the boards for the argument they find most convincing. Choose one to consolidate — ideally one that uses the language of the definitions clearly. Students copy the chosen argument into their notes.

The argument in student-friendly language

∠ABC and ∠ABD are a linear pair → ∠ABC + ∠ABD = 180°. ∠ABD and ∠CBE are a linear pair → ∠ABD + ∠CBE = 180°. Both equal 180°, so ∠ABC = 180° − ∠ABD and ∠CBE = 180° − ∠ABD. Since they equal the same quantity, ∠ABC = ∠CBE.

Students must understand and be able to explain this argument, not just copy it. Ask: what was the key move? (Both angles equal 180° minus the same thing.)

Narrative Arc — End of Lesson 1.4 — after the vertical angles proof

"Look at what just happened. You proved something. Not because I told you it was true. Not because a textbook printed it. You proved it because you reasoned from definitions you built yourselves and arrived somewhere you could defend."

"That is the difference between this class and a class where facts get handed to you. We will keep doing this all year. The arguments will get longer and more formal. The tools will accumulate. But the act — conjecturing, testing, arguing from definitions — is the same act you just performed. Remember this moment when proof writing feels hard later on. You already know how to do this."

Students make notes on:

- Angle naming: three-point notation with examples; when one-letter naming fails
- Definitions: linear pair, vertical angles, supplementary, complementary, adjacent — each with diagram
- The vertical angles argument written in students' own words (from the chosen group proof)
- A note on what was just proved and why it counts as a proof

Connection Points	• Draws from Unit 0: the vertical angles argument is the Unit 0 ‘convincing argument’ practice applied to geometric objects for the first time • Feeds into Unit 2: linear pairs and supplementary angles are used as given information in transversal proofs • Feeds into Unit 3: the vertical angles theorem is cited as a known result in proofs from Lesson 3.3 onward