

Convincing Arguments

Unit 0: Problem Solving and Mathematical Habits • Day 3 of 4

At a Glance

Pacing: ~40 min thinking task + gallery walk • ~20 min consolidation (mathematical thinking framework) • ~30 min class policies conversation

Materials: Boards and markers • Magnetic grids (one per group if available)

- Group work rubric posted at workstations

Mathematical habits introduced today: Conjecturing • Justifying • Convincing (yourself → your group → your peers → the world)

Group assessment: Randomly select 3 groups to assess using the group work rubric from Lesson 0.2. All groups self-assess. Compare results with those 3 groups.

OBJECTIVES

By the end of this lesson students:

- Understand the distinction between showing a path exists (one example suffices) and showing no path can exist (an argument is required).
- Can describe the full mathematical thinking framework: get started, specialize, organize, generalize, test, conjecture, justify/convince, reflect.
- Have experienced, from the inside, why failed attempts are not the same as a proof of impossibility.

THINKING TASK

Chessboard Paths (≈35 min)

Source: adapted from Thinking Visually by James Tanton

Prompt

On a 5×5 grid, pick any square to start. You can move up, down, left, or right, but not diagonally. Can you visit every square exactly once without repeating a square?

Give each group a magnetic grid if available. Let students try different paths freely. As groups make discoveries, ask:

- How do you know a path is impossible from that square?
- Is one failed attempt enough to prove that something is impossible?
- How many examples do you need to test?
- Is there another way to convince others that a square won't work?

The big idea to surface

To show a path exists: one example is enough.

To show no path can exist: an argument is required — not just failed attempts.

The mathematical argument (for teacher reference)

The 5×5 grid has 25 squares. Color them like a checkerboard. Any path must alternate colors with every move. Since 25 is odd, the path must end on the same color it started on. Starting from a white square: moves 1, 3, 5, ... 23 land on colored squares; move 24 lands on white; but only one white square remains and the final move cannot reach it. Therefore no Hamiltonian path exists from a white corner square. Starting from a colored center square: a path does exist (one example suffices).

Extension ladder: Try 6×6, 7×7, 9×9 grids. Can they predict which squares work without testing all of them? Is there a symmetry in the pattern of working squares?

CONSOLIDATION — TWO PARTS

Part 1: The thinking framework. Share the complete list with students now that they have experienced all the moves across three days. Name each step using the tasks from Days 1, 2, and 3 as examples.

Mathematical thinking framework — display and name each step (from *Thinking Mathematically* by Mason, Burton, Stacey)

Get started: try something. Draw, guess, rephrase.

Specialize: try specific cases.

Organize: look for structure or order in your cases.

Generalize: use variables or patterns.

Test: try to break your rule. Look for counterexamples.

Conjecture: make a claim or general rule.

Justify/Convince: can you prove it? Can you explain why? Convince yourself → your group → your peers → the world.

Reflect: what worked, what didn't, and what's next?

Part 2: Class policies. Spend time here on course policies, the grading rubric, and reassessment procedures. This is the right moment — students now have context for why the class is structured the way it is.

Note on grading policy — adapt to your context

Our course used a rubric-based system: formative work 5%, daily quizzes 10%, summative assessments 85%. Rubric levels: 7 (exceeds), 6 (meets), 5 (minor clarification needed), 1 (not yet proficient). Reassessment was available because learning is a process and grades should reflect understanding when it is reached.

Teachers should spend genuine time on the purpose of grades, homework, and reassessment — in your own words, aligned with your own beliefs. Students who understand why the policies exist are more likely to engage with them honestly.

 **Narrative Arc** — End of the consolidation — name the arc across all three days

“Day 1 we learned to try specific examples. Day 2 we learned to organize them and use variables to generalize. Today we asked: what does it take to actually convince someone? Not just that it seems to be true — but that it must be. That question is going to follow us all year.”

 **Students make notes on:**

- Mathematical thinking framework: all eight steps, with one example from the unit tasks for each
- The key distinction: one example proves existence; an argument is required to prove impossibility

Feeds into

- Unit 2, Lesson 2.1: the circle task is a direct callback to the existence vs. impossibility distinction from today
- Unit 3: the entire unit on proof is the formalization of the ‘convincing argument’ question raised here