

Check Your Understanding 2.4

Covers all rubric rows introduced through 2.4. Students begin in class.

Basic	Intermediate	Advanced
<i>Law of Detachment — Basic</i> 1. What can you conclude? Name the law. <i>a. If a person is a teacher, they work at school. Maria is a teacher. $\therefore$?</i> <i>b. If a figure is a square, it has four equal sides. ABCD is a square. $\therefore$?</i>	<i>Law of Detachment — Identify the Error</i> <i>2. Maria earns extra money. If a person works overtime, they earn extra money. $\therefore$ Maria worked overtime.</i> a. Valid or invalid? b. Name the fallacy if invalid. c. Rewrite so it is valid.	<i>Law of Detachment — Construct</i> 3. Write your own example of the law of detachment used correctly. Then change one part to create an invalid argument and explain what went wrong.
<i>Law of Syllogism — Basic</i> 4. What can you conclude? <i>If a student misses class $\rightarrow$ they miss notes.</i> <i>If they miss notes $\rightarrow$ they fail the quiz.</i> a. What can we conclude if a student misses class? b. What law was applied?	<i>Law of Syllogism — Intermediate</i> 5. State the law of syllogism in your own words and show an example using symbols. 6. Write a 3-step logical chain about a real-world situation. It must be valid but does not need to be sound.	<i>Law of Syllogism — New Puzzle</i> 7. Solve this puzzle: <i>(a) All members of the astronomy club own a telescope.</i> <i>(b) No one who owns a telescope has ever missed a meteor shower.</i> <i>(c) Anyone who has missed a meteor shower is not a serious observer.</i> Write each premise as a conditional. Use contrapositives where needed. Chain them to reach a conclusion about astronomy club members and serious observers.

Basic	Intermediate	Advanced
<i>Denying the Consequent — Basic</i> 8. If someone is a doctor, they have a medical degree. Maya does not have a medical degree. $\therefore$? a. What can you conclude? b. What law was applied?	<i>Disjunctive Syllogism — Intermediate</i> 9. I told my counselor I would take either Spanish 1 or French 3. She said French 3 does not fit my schedule. $\therefore$? a. What can you conclude? b. What law applies? c. Write your own example of disjunctive syllogism.	<i>Denying the Consequent — Advanced</i> 10. Use a Venn diagram to explain why denying the consequent is valid. 11. If it is sunny, the picnic will happen. The picnic did not happen. a. What can you conclude? Name the law. b. Draw a Venn diagram to support your answer.
<i>Spiral — Evaluate Arguments</i> 12. Identify each as valid, sound, or invalid. If invalid, name the fallacy. a. All reptiles are cold-blooded. A snake is a reptile. $\therefore$ A snake is cold-blooded. b. If it is cold-blooded, then it is a reptile. A snake is cold-blooded. $\therefore$ A snake is a reptile.	<i>Spiral — Conditional Statements</i> 13. Write the contrapositive and state its truth value: a. If a number is divisible by 9, then it is divisible by 3. b. If it is a dolphin, then it is a mammal.	<i>Extension — Geometry Preview</i> 14. Consider: If two lines are parallel $\rightarrow$ alternate interior angles are congruent. If alternate interior angles are congruent $\rightarrow$ the lines are parallel. a. Is this biconditional? How do you know? b. How might the law of syllogism be used when we prove things in geometry? c. What would it mean for geometry to be 'one large application of the law of syllogism from 10 premises'?

CYU 2.4 — Answer Key

1a. Maria works at school. Law of Detachment.

1b. ABCD has four equal sides. Law of Detachment.

2a. Invalid. **2b.** Affirming the consequent. **2c.** If a person works overtime, they earn extra money. Maria worked overtime. $\therefore$ Maria earns extra money.

3. Answers vary. Valid: $p \rightarrow q$, $p \therefore q$. Invalid version must change structure (not just content).

4a. If a student misses class, they fail the quiz. **4b.** Law of Syllogism.

5. If $p \rightarrow q$ and $q \rightarrow r$, then $p \rightarrow r$. The consequent of the first must match the antecedent of the second.

6. Answers vary. Check: three steps, each conclusion feeds into the next hypothesis.

7. Conditionals: (a) astronomy club member $\rightarrow$ owns telescope. (b) owns telescope $\rightarrow$ has not missed meteor shower. (c) missed meteor shower $\rightarrow$ not a serious observer. Contrapositive of (c): serious observer $\rightarrow$ has not missed meteor shower (not needed directly). Chain runs forward: member $\rightarrow$ owns telescope [a] $\rightarrow$ not missed meteor shower [b]. Contrapositive of (c): not missed meteor shower $\rightarrow$ is a serious observer. Full chain: member $\rightarrow$ owns telescope $\rightarrow$ not missed meteor shower $\rightarrow$ is a serious observer. $\therefore$ All members of the astronomy club are serious observers.

8a. Maya is not a doctor. **8b.** Denying the Consequent (Modus Tollens).

9a. I will take Spanish 1. **9b.** Disjunctive Syllogism. **9c.** Answers vary.

10. Venn diagram: p-circle inside q-circle. If x is not in q, it cannot be in p (since p is entirely inside q). That is $\sim q \rightarrow \sim p$ — the contrapositive.

11a. It is not sunny. Denying the Consequent.

12a. Sound — valid structure (law of detachment), true premises.

12b. Invalid — affirming the consequent. Being cold-blooded does not guarantee being a reptile (fish are also cold-blooded).

13a. If not divisible by 3, then not divisible by 9. TRUE. **13b.** If not a mammal, then not a dolphin. TRUE.

14a. Yes — biconditional. Both directions are true.

14b. Each proof step chains a previously established result into a new conclusion. Law of detachment and syllogism are the mechanisms of every geometric proof.

14c. Every theorem in Euclidean geometry ultimately traces back to Euclid's 5 postulates and 5 common notions through chains of conditional reasoning. The law of syllogism is what makes each link in the chain possible.