Lesson 7.6 — Teacher Reference Notes

The Law of Sines

Unit 7: Trigonometry • Days 9–10 • Day 1: Area, Altitudes, and Deriving the Law • Day 2: Mount Everest

At a Glance	
	Day 1 pacing: Phase 1: Thinking task — group cards (~30 min) • Phase 2: Consolidation — derive Law of Sines (~15 min) • Phase 3: Structured note-making (~10 min) • Phase 4: CYU start (~15 min or homework) **Day 2 pacing:** Open: Why can't we use right-triangle trig on Everest? (~10 min) • Thinking task: develop a plan (~20 min) • Reveal numbers and solve (~20 min) • Error discussion + closing message (~10 min) • CYU finish if not completed **Materials:** Group cards 1–8 printed and cut • Boards and markers • Calculators • Everest numbers held by teacher until groups have a viable plan **Nav instrument rows active:** Row 6 (Law of Sines) activates today — all three levels **Year 1 highlight:** Every group used trig to write the altitude independently without prompting. The most common error was assuming the altitude bisects the base — this led to one of the best class discussion of the year. Keep this conversation ready. **Day 2: Hold the numbers:** Do not reveal the elevation angles or baseline until groups have presented a viable plan. The planning stage is the lesson.

OBJECTIVES

By the end of this lesson students can:

- Find the area of a triangle given two sides and the included angle: Area = ½ab·sin C.
- Derive the Law of Sines algebraically from the three equal area expressions.
- Apply the Law of Sines to find unknown sides and angles in non-right triangles.
- Apply the Law of Sines in a multi-step problem requiring linear pairs and triangle angle sum.
- Distinguish between measurement error and mathematical error.

Day 1 — Area, Altitudes, and the Law of Sines

Narrative Arc — Open Day 1 — before distributing cards

"You have spent this unit working with right triangles. Every tool you have uses a right angle somewhere. Today the question changes: what if there is no right angle? Any triangle can be split into right triangles by dropping an altitude. Today you discover what that gives you."

PHASE 1 — THINKING TASK: GROUP CARDS (≈30 MIN)

Distribute one card per group. Each group has a different fully-labeled triangle. All triangles are acute.

> **Task**
>
> **Your card gives you a complete triangle. Draw it on your board and label everything. Find the area. Then find it again a completely different way using different sides. Then find it a third way. Extension: you now have three expressions that all equal the area. Write them in general form using letters a, b, c, A, B, C. What do you notice?**

Group cards reference:

Card 1	Card 2	Card 3	Card 4
A=40°, B=60°, C=80° a=8, b=10.78, c=12.26 Area ≈ 42.46 cm²	A=55°, B=65°, C=60° a=11, b=12.17, c=11.63 Area ≈ 57.97 cm²	A=35°, B=75°, C=70° a=6, b=10.10, c=9.83 Area ≈ 28.48 cm²	A=50°, B=70°, C=60° a=9, b=11.04, c=10.17 Area ≈ 43.02 cm²
Card 5	**Card 6**	**Card 7**	**Card 8**
A=45°, B=65°, C=70° a=10 Area ≈ 60.22 cm²	A=60°, B=70°, C=50° a=12 Area ≈ 59.85 cm²	A=40°, B=75°, C=65° a=8 Area ≈ 43.58 cm²	A=55°, B=60°, C=65° a=11 Area ≈ 57.97 cm²

> ⚠️ **What to watch for**
>
> **The key move:** every group should independently drop an altitude to write h = b·sin A (or equivalent). If a group stalls, ask: ‘How would you find the height of this triangle using trig?’
>
> **The perpendicular bisector error:** students frequently assume the altitude bisects the base. It only does so if the triangle is isosceles. When this appears: ‘Before you can use the conclusion of a theorem, what must be true?’ This is the lesson from Unit 2 applied geometrically.
>
> **The cosine variation:** if a group writes the area using the top angle and cosine, this is not wrong. It is worth discussing in consolidation.
>
> **The extension:** groups that reach the general form ½bc·sin A = ½ac·sin B = ½ab·sin C have done the hardest inductive work. Ask them to present.

PHASE 2 — CONSOLIDATION: DERIVING THE LAW OF SINES (≈15 MIN)

Run the derivation at the board together. Ask a group that reached the general form to present first, then complete the algebra.

> **Deriving the Law of Sines**
>
> Three ways to express the area of triangle ABC, dropping altitude h from each vertex:
>
> **Area = ½·b·c·sin A = ½·a·c·sin B = ½·a·b·sin C**
>
> All three equal the same area. Divide every term by ½abc:

sin A / a = sin B / b = sin C / c

Name the distinction explicitly: every group found the same ratio structure in their own triangle — that was inductive evidence. The algebraic derivation from a general triangle is deductive proof. This is what Unit 2 was preparing students for.

PHASE 3 — STRUCTURED NOTE-MAKING (≈10 MIN)

✎ **Students make notes on:**

- Area formula: Area = ½ab·sin C (and the two equivalent forms using the other angle pairs) — with a labeled diagram showing which sides and angle are used
- The Law of Sines: sin A/a = sin B/b = sin C/c — written as both the ratio form and the cross-multiplied form
- When to use it: at least one known angle-side pair is required
- The altitude as the proof tool: diagram showing how the altitude h = b·sin A creates the right triangle

Note-making prompt to post on the board

Record: the area formula, the Law of Sines, and a worked example using your group's card. Your example should show every step from identifying the known information to the final answer.

PHASE 4 — CYU PROBLEMS (≈15 MIN OR HOMEWORK)

Q4 — the ambiguous case (SSA)

Q4 gives A = 44°, a = 13, b = 17. This is an SSA configuration — the ambiguous case. Two triangles are possible: B ≈ 65.5° and B ≈ 114.5°. If you have not covered the ambiguous case, accept either answer or name it briefly: 'When you find an angle using inverse sine, there is sometimes a second valid answer.' The Law of Cosines (Lesson 7.7) resolves this ambiguity for the SAS and SSS cases.

Day 2 — How Do We Know How Tall Everest Is?

▶ **Narrative Arc** — Open Day 2 — the question before the question

"We now know how to measure inaccessible heights using angles of elevation and a horizontal distance. So why not just do that for Everest? Stand at the base. Measure the angle to the summit. Measure the horizontal distance. Done. What's the problem?"

The problem — develop this with students before releasing the task

You cannot stand at the base of Everest. The terrain is the Himalayas — glaciers, ridges, no flat ground. You cannot measure the horizontal distance to the base because you cannot access the base. You cannot set up a right triangle with a measurable horizontal leg.

The solution: find flat land some distance away where the summit is still visible. Two points on that flat land, a known distance apart, each measure the elevation angle to the summit. That baseline and those two angles are enough. Ask: what tools from this unit could work here?

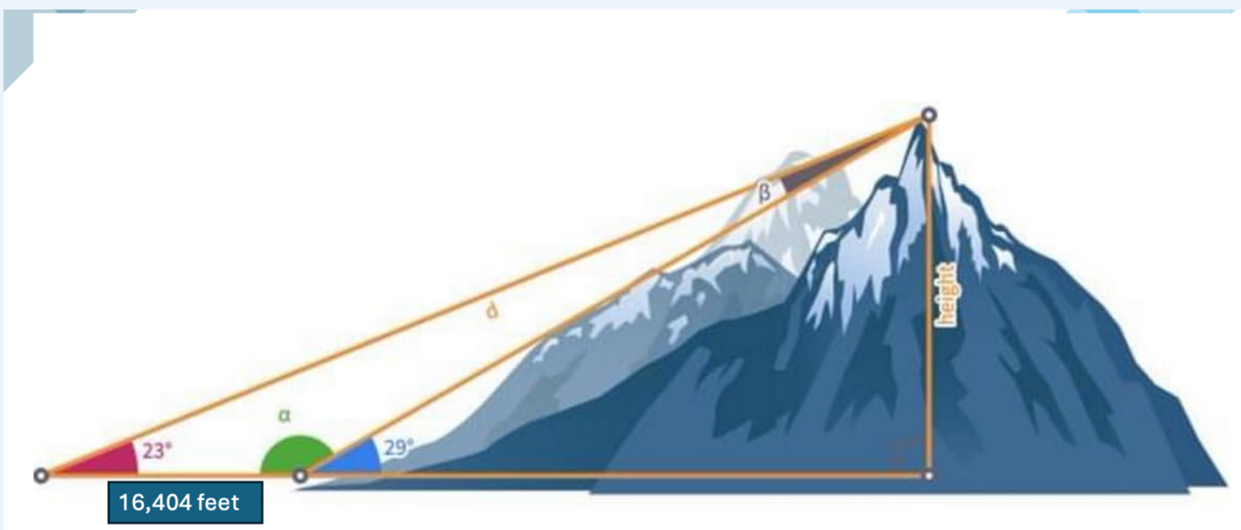
DAY 2 — THINKING TASK: DEVELOP A PLAN (≈20 MIN)

Task prompt

Imagine flat land some distance from the mountain where the summit is still visible. Two surveyors at different positions on that flat land measure the elevation angle to the summit. The baseline between them is known. Design a method. When you have a plan, present it — then I will give you the actual survey numbers."

For teacher reference:

Survey numbers (reveal once groups have a viable method): Point A elevation angle 23°, Point B elevation angle 29°, baseline AB = 16,404 feet.



Solution: Angle at B interior = 180°−29° = 151° (linear pair). Angle at P = 180°−23°−151° = 6°. Law of Sines: AP = 16,404 × sin(151°)/sin(6°) ≈ 76,120 ft. Height = 76,120 × sin(23°) ≈ 29,728 ft. Actual: 29,032 ft. Error: 2.4% from whole-degree angle precision.

⚠️ **Guiding questions if groups are stuck**

On which triangle to use: 'You have two observation points and the summit. What triangle does that form? What do you know about it?'

On the interior angle at B: ‘The elevation angle is measured from horizontal. The interior angle of your triangle at B is different. Draw horizontal at B and think about the angle between line BA and line BP.’

On moving from the Law of Sines to height: ‘Once you know the slant distance AP, what kind of triangle gives you the height?’

The Everest Solution — Full Worked Calculation (Reveal after groups have a plan)

Given: Point A (far): elevation angle to summit = 23° Point B (near): elevation angle to summit = 29° Baseline AB = 16,404 feet

Step 1 — Find the interior angles of triangle ABP (P = summit)

∠ at A = 23° (elevation angle; the horizontal at A is the base of this angle)

∠ at B: The elevation angle 29° is measured from horizontal at B toward the summit. But the interior angle of triangle ABP at B is the angle between line BA and line BP. Since BA and the horizontal at B point in opposite directions, the interior angle at B = 180° − 29° = 151°.

∠ at P = 180° − 23° − 151° = 6° (triangle angle sum)

Step 2 — Law of Sines to find AP

AP / sin(151°) = AB / sin(6°)

AP = 16,404 × sin(151°) / sin(6°) ≈ 16,404 × 0.4848 / 0.1045 ≈ 76,120 feet

Step 3 — Right triangle trig to find height

Height = AP × sin(23°) ≈ 76,120 × 0.3907 ≈ 29,742 feet

Actual height: 29,032 feet. Our answer: ≈ 29,742 feet. Error: ≈ 2.4%.

The 2.4% error comes from using whole-degree angles. Surveyors measure to arc-seconds (1° = 3,600 arc-seconds). Our calculation is mathematically correct — the error is a measurement precision issue, not a mathematical one. Name this distinction explicitly.

DAY 2 — THE CLOSING MESSAGE

Say this — after revealing the answer

‘We built knowledge on knowledge on knowledge. Because we derived a collection of facts about abstract shapes and developed as problem solvers, we were able to tackle something massive and seemingly impossible. When we were proving theorems about parallel lines and triangle congruence and similar figures, we were not thinking: this is how we will calculate Everest’s height. If we had not learned those things, we would have found this problem impossible.’

‘The 2.4% error is not a mathematical mistake. It is a measurement precision issue. Our mathematics was exact. The angle measurements were rounded to whole degrees. Surveyors measure to arc-seconds — one degree contains 3,600 arc-seconds. That is why the actual measurement is more precise than ours, not because our method was wrong.’

Connection Points	• Draws from Unit 2: the inductive/deductive distinction is named explicitly in the consolidation — every group found the ratio pattern (inductive), the algebraic derivation proved it (deductive) • Draws from Lesson 4.3: the altitude from the apex of an isosceles triangle bisects the base — the Year 1 error (altitude bisects the base of any triangle) is corrected by applying this theorem precisely • Draws from Lesson 6.4: the mirror and shadow methods (indirect measurement with right triangles) are the predecessors that make the Everest problem feel impossible and necessary • Draws from Lessons 7.2 and 7.3: right triangle trig is used in the final step of the Everest problem to extract the height from AP × sin(23°) • Feeds into Lesson 7.7: CYU Q8 names the limitation — Law of Sines requires an angle-side pair. Law of Cosines handles SSS and SAS.