

Check Your Understanding 8.3

Show all work and reasoning. Spiral problems revisit earlier rows — use them to keep those ideas sharp.

Basic	Intermediate	Advanced
Row 1 Spiral — What is a circle? → <i>Defining the object</i>		
<i>Basic — Spiral</i> 1. A circle passes through (7, 0) with center (3, 0). Find the radius and write the equation.	<i>Intermediate — Spiral</i> 2. For that circle, is the point (3, 4) on, inside, or outside it? Justify.	<i>Advanced — Spiral</i> 3. Does that circle overlap, touch once, or miss the circle $x^2 + (y - 2)^2 = 9$? Justify using the distance between centers and the radii.
Row 2 Spiral — All circles are similar → <i>Why proportional reasoning works</i>		
<i>Basic — Spiral</i> 4. Circle P (radius 3) and Circle Q (radius 7). Describe how to map P onto Q in two steps.	<i>Intermediate — Spiral</i> 5. Circle P has circumference 6π . Using the similarity ratio (not a fresh calculation), find Circle Q's circumference.	<i>Advanced — Spiral</i> 6. Circle P has a 60° sector with arc length π ; Circle Q has a 60° sector. Without the arc-length formula, explain why Q's arc is $7\pi/3$, and name the property that makes the argument valid.
Arc length and sector area → <i>Applying proportional reasoning</i>		
<i>Basic</i> 7. A circle has radius 5 and a central angle of 72° . (a) What fraction of the circle is the sector? (b) Find the arc length. (c) Find the sector area.	<i>Intermediate</i> 8. A circle has radius 12. (a) A sector has central angle 150° — find the arc length and sector area. (b) A different sector of the same circle has arc length 6π — find the central angle.	<i>Advanced</i> 9. A sector has central angle 240° and sector area 48π . (a) Find the radius. (b) Find the arc length. 10. A sprinkler rotates through 110° and reaches 15 ft. (a) What area does it cover? (b) What is the arc length at maximum reach? (c) To cover exactly 200 ft^2 at the same angle, what reach is needed (exact)?
Algebra Spiral — Exponent Rules		
<i>Simplify. Write every answer with positive exponents.</i> (a) $(2x^3)^4$ (b) $x^6 \div x^{-2}$ (c) $(4a^2b)(3ab^3)$ (d) $(m^3)^{-2}$		

Answers — CYU 8.3

Row 1 Spiral — What is a circle?

1. $r = 4$; $(x - 3)^2 + y^2 = 16$.
2. $(3 - 3)^2 + 4^2 = 16 = r^2$, so $(3, 4)$ is on the circle.
3. Second circle: center $(0, 2)$, $r = 3$. Center distance $= \sqrt{13} \approx 3.6$. Since $|4 - 3| = 1 < \sqrt{13} < 4 + 3 = 7$, the circles overlap (two intersection points).

Row 2 Spiral — All circles are similar

4. Shift P's center onto Q's center, then dilate by scale factor $7/3$.
5. $6\pi \cdot (7/3) = 14\pi$. Lengths scale by the similarity ratio.
6. Arc is a length, so it scales by $7/3$: $\pi \cdot 7/3 = 7\pi/3$. All circles are similar, so sectors with the same central angle are similar figures; their arc lengths are in the ratio of the radii.

Arc length and sector area

7. (a) $72/360 = 1/5$. (b) $(1/5) \cdot 2\pi(5) = 2\pi$ cm. (c) $(1/5) \cdot \pi(25) = 5\pi$ cm².
8. (a) Arc $= 10\pi$ cm, area $= 60\pi$ cm². (b) $6\pi = (\theta/360) \cdot 24\pi \rightarrow \theta = 90^\circ$.
9. (a) $(2/3)\pi r^2 = 48\pi \rightarrow r^2 = 72 \rightarrow r = 6\sqrt{2}$. (b) $(2/3) \cdot 2\pi(6\sqrt{2}) = 8\pi\sqrt{2}$.
10. (a) $(11/36)\pi(225) = 275\pi/4 \approx 216.4$ ft². (b) $(11/36) \cdot 2\pi(15) = 55\pi/6 \approx 28.8$ ft. (c) $(11/36)\pi r^2 = 200 \rightarrow r = \sqrt{(7200/(11\pi))}$.

Algebra Spiral — Exponent Rules

- (a) $16x^{12}$
- (b) x^8
- (c) $12a^3b^4$
- (d) $1/m^6$