

## Check Your Understanding 5.5

| Basic                                                                                                                                                                                                                                                                                                    | Intermediate                                                                                                                                                                                                                                                                                                                      | Advanced                                                                                                                                                                                                                                                                                                                   |
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| <b>Row 3 Spiral — Special Parallelograms</b><br>→ Adding constraints, inheriting results                                                                                                                                                                                                                 |                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                            |
| <i>Basic — Spiral</i><br>1. In rectangle ABCD, diagonal $AC = 7x - 3$ and diagonal $BD = 4x + 9$ . Find $x$ and the length of each diagonal.<br><br>2. Name the special parallelogram whose diagonals bisect the vertex angles.                                                                          | <i>Intermediate — Spiral</i><br>3. In rhombus PQRS, $\angle PQR = 78^\circ$ . Find all four angles and the angles formed where the diagonals intersect.                                                                                                                                                                           | <i>Advanced — Spiral</i><br>4. A student uses coordinates to confirm that a quadrilateral has equal diagonals. They conclude it must be a rectangle. Is this correct? What additional check is required? Write the complete set of conditions needed to classify a quadrilateral as a rectangle using coordinate methods.  |
| <b>Row 4 Spiral — Trapezoids and Kites</b><br>→ Structure outside the parallelogram family                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                            |
| <i>Basic — Spiral</i><br>5. In isosceles trapezoid ABCD ( $AB \parallel CD$ ), $\angle BCD = 71^\circ$ . Find all four angles.<br>6. In kite EFGH with $EF = EH$ and $GF = GH$ , the axis diagonal $EG = 15$ . $EJ = 4$ . Find GJ.                                                                       | <i>Intermediate — Spiral</i><br>7. Isosceles trapezoid PQRS has bases $PQ = 10$ and $RS = 18$ , and legs $PS = QR = 5$ . Find the height and area of the trapezoid.                                                                                                                                                               | <i>Advanced — Spiral</i><br>8. A quadrilateral ABCD has vertices $A(-3, 0)$ , $B(3, 0)$ , $C(2, 3)$ , $D(-2, 3)$ . Classify it using coordinate methods. Then verify that the diagonals are congruent using distance.                                                                                                      |
| <b>Coordinate Methods</b><br>→ A second proof system                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                            |
| <i>Basic</i><br>9. Quadrilateral ABCD has vertices $A(0, 0)$ , $B(4, 0)$ , $C(4, 3)$ , $D(0, 3)$ . Use slope to show both pairs of opposite sides are parallel. Use distance to show all four sides are not equal. Name the most specific classification and explain what additional check confirmed it. | <i>Intermediate</i><br>10. Quadrilateral PQRS has vertices $P(-3, 1)$ , $Q(0, 4)$ , $R(4, 1)$ , $S(1, -2)$ . Classify PQRS as specifically as possible. Show every check your classification requires. State which conditions are sufficient and which additional checks you performed to rule out more specific classifications. | <i>Advanced</i><br>11. Three vertices of a rectangle are $A(-2, 3)$ , $B(4, 3)$ , and $C(4, -1)$ . Find the fourth vertex D. Write a complete coordinate proof that ABCD is a rectangle — not just that it has the properties of a rectangle, but that it satisfies the definition: a parallelogram with four right angles |
| <b>Algebra Spiral</b><br>Solve systems of two equations                                                                                                                                                                                                                                                  | <i>Algebra Spiral</i><br>Solve each system. Show your method.<br>(a) $x + 2y = 8$ and $3x - y = 1$<br>(b) $y = -2x + 5$ and $4x + 3y = 10$                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                            |

**Answers — CYU 5.5****Row 3 Spiral**

1.  $7x-3=4x+9 \rightarrow x=4$ . Each diagonal = 25.
2. Rhombus (and square).
3.  $\angle PQR=78^\circ \rightarrow \angle PSR=78^\circ$ .  $\angle QPS=\angle QRS=102^\circ$ . Diagonals bisect:  $\angle PQE=\angle RQE=39^\circ$ . Intersection angles all  $90^\circ$ .
4. No — not sufficient. Equal diagonals alone only confirms rectangle if it is already known to be a parallelogram. Need: (1) both pairs of opposite sides parallel, (2) at least one right angle (or equivalently, perpendicular adjacent sides), (3) then diagonals equal follows. The full check: show parallelogram via slope, then perpendicular adjacent sides via slope product = -1.

**Row 4 Spiral**

5.  $\angle BCD=71^\circ \rightarrow \angle ADC=71^\circ$ .  $\angle DAB=\angle ABC=109^\circ$ .
6.  $GJ = EG - EJ = 15 - 4 = 11$ .
7. Overhang each side =  $(18-10)/2 = 4$ . Height =  $\sqrt{(5^2-4^2)} = 3$ . Area =  $\frac{1}{2}(10+18)(3) = 42$ .
8. Slope  $AB=0$ , Slope  $DC=0 \rightarrow AB \parallel DC$ . Slope  $AD=3$ , Slope  $BC=-3 \rightarrow$  not parallel  $\rightarrow$  exactly one pair  $\rightarrow$  trapezoid.  $AD=BC=\sqrt{(1+9)}=\sqrt{10} \rightarrow$  isosceles trapezoid.  $AC=\sqrt{[(2-(-3))^2+(3-0)^2]}=\sqrt{34}$ .  $BD=\sqrt{[(-2-3)^2+(3-0)^2]}=\sqrt{34}$ . Diagonals equal  $\checkmark$ .

**Coordinate Methods**

9. Slope  $AB=0$ ,  $DC=0 \rightarrow \parallel$ . Slope  $BC=\text{undefined}$ ,  $AD=\text{undefined} \rightarrow \parallel$ .  $AB=4$ ,  $BC=3$  — unequal  $\rightarrow$  not rhombus. Perpendicular slopes  $\rightarrow$  right angles  $\rightarrow$  rectangle.
10. Slope  $PQ=1$ ,  $SR=1 \rightarrow \parallel$ . Slope  $QR=-3/4$ ,  $PS=-3/4 \rightarrow \parallel$ . Parallelogram.  $PQ=3\sqrt{2}$ ,  $QR=5 \rightarrow$  unequal  $\rightarrow$  not rhombus. Slope product =  $(1)(-3/4) = -3/4 \neq -1 \rightarrow$  not rectangle. Parallelogram only.
11.  $D=(-2,-1)$ . Proof: Slope  $AB=0$ ,  $DC=0 \rightarrow \parallel$ . Slope  $BC=\text{undefined}$ ,  $AD=\text{undefined} \rightarrow \parallel$ . Parallelogram. Slope  $AB=0 \perp$  Slope  $BC$  (undefined)  $\rightarrow$  right angle at B  $\rightarrow$  rectangle. ■

**Algebra Spiral**

- (a)  $x = 2$ ,  $y = 3$ .
- (b)  $x = 5/2$ ,  $y = 0$ .