

When Is Similarity Guaranteed?

Unit 6: Similarity • Day 3 of unit

At a Glance

Pacing: Framing + Scenario 1 (~10 min) • Scenarios 2–6 board work (~35 min) • Consolidation: three conjectures + epistemological note (~15 min) • Note-making + CYU (~20 min)

Materials: • Rulers and protractors for every group (accuracy matters) • Calculators

Nav instrument rows active: Row 2 (Triangle Similarity Shortcuts) — all three levels activate today

Scenarios: Read each scenario aloud. Groups work until they either find a counterexample or are convinced they cannot. You decide when to move to the next scenario.

Key norm to establish before beginning: No number of confirming examples proves a statement true. One valid counterexample proves it false. Both kinds of results are valuable.

OBJECTIVES

By the end of this lesson students can:

- Explain why one or two conditions are insufficient to guarantee triangle similarity, with a counterexample for each failing case.
- State AA~, SAS~, and SSS~ as sufficient conditions for triangle similarity and explain why each works.
- Distinguish between failing to find a counterexample (evidence) and a proof (certainty).
- Use AA~, SAS~, or SSS~ to justify triangle similarity in diagrams.
- Write similarity statements with correct correspondence and extract proportional side relationships.

The epistemological norm — establish before any scenario

Say explicitly before Scenario 1:

‘In Unit 3, we discovered shortcuts for congruence by finding which conditions were sufficient and which weren’t. Today we do the same for similarity. The method is the same: try to break it. If you can find two triangles that satisfy the condition but are not similar, the condition fails. If you cannot — despite serious effort — that’s evidence it might be sufficient. But failing to find a counterexample is not a proof. It is a conjecture. We will accept the successful conditions today and note that they can be proven.’

► **Narrative Arc** — Open class — the Unit 3 parallel

“In Unit 3 you discovered that proving triangle congruence requires six conditions — three side pairs and three angle pairs. But you also discovered shortcuts: SSS, SAS,

ASA, AAS. Fewer conditions, same certainty. Today we ask the same question for similarity. Two similar triangles also need six conditions: three equal angle pairs and three proportional side pairs. Do we need all six? Or are there shortcuts?"

THE SIX SCENARIOS — READ ALOUD, ONE AT A TIME

Give each scenario as a verbal prompt. Groups work at boards with rulers and protractors. When they find a counterexample, ask them to hold it up and explain it to the class. When they fail to find one, ask them to write their conjecture on the board before you move on.

#	Condition	What happens	Sufficient?	Teacher move
1	One pair of corresponding angles congruent	Groups find counterexamples immediately.	No	Let them confirm and move on quickly. Reinforce: one angle gives a family of shapes, not a unique shape.
2	Two pairs of corresponding angles congruent (AA)	Groups struggle to find counterexamples. Most cannot.	Yes — but...	Do not confirm yet. Name the tension: 'Failing to find a counterexample is evidence but not proof. Write your conjecture.'
3	One pair of corresponding sides in proportion	Counterexamples found immediately.	No	Reinforce: one ratio alone says nothing about shape. A 1:2 ratio applies to infinitely many triangles.
4	Two pairs of corresponding sides in proportion	Most groups find counterexamples (e.g., an isosceles and a scalene triangle can share two side ratios).	No	Name it: SS is not enough. The included angle is what pins the shape.
5	Two pairs of sides in proportion AND included angle congruent (SAS~)	Groups fail to find counterexamples.	Yes — but...	Same move as Scenario 2. Conjecture recorded, not yet confirmed.
6	All three pairs of sides in proportion (SSS~)	Groups fail to find counterexamples.	Yes — but...	Same move. Three conjectures are now on the boards.

⚠ Critical teacher moves for Scenarios 2, 5, and 6

When groups report they cannot find a counterexample for AA~, SAS~, or SSS~ — do not confirm. Say:

'You haven't found a counterexample. That's different from knowing there isn't one. Write your conjecture: "We believe this condition is sufficient because we could not construct a counterexample."

This is a deliberate pedagogical choice. Students have not proved these shortcuts. They have strong evidence. The consolidation names the distinction explicitly and accepts the shortcuts as postulates — just as the course did with SSS, SAS, and ASA in Unit 3.

Scenario 4 note — the counterexample to construct

Two sides in proportion but no angle condition (SS). A clean counterexample: let triangle 1 have sides 3, 4, 6 and triangle 2 have sides 6, 8, 9. The ratios $3/6 = 4/8 = 1/2$ but $6/9 \neq 1/2$ — so not SSS~. But the point is simpler: take any scalene triangle and an isosceles triangle that happen to share two side ratios. Their shapes will differ.

If groups struggle to find a counterexample here: suggest trying one isosceles and one scalene triangle where two sides are in proportion. The third side ratio will differ.

CONSOLIDATION — THREE PARTS

Part A — Name the three conjectures

Ask three groups to read their conjectures aloud for AA~, SAS~, and SSS~. Write all three on the board. Then say:

‘These three conjectures are correct. They can each be proven from the definition of similarity and from theorems about parallel lines and angle sums. The proofs are not trivial. Rather than spend the rest of this unit proving the shortcuts, we are going to accept them as postulates and use them as tools — just as we did with SSS, SAS, and ASA in Unit 3. We name the choice honestly: these are accepted without proof here.’

Part B — AA~ deserves special attention

Name the reason AA~ works more explicitly than the others. Ask: ‘If we know two angles of a triangle, what do we know about the third?’

Students should say: the triangle angle sum forces it. If $\angle A = \angle D$ and $\angle B = \angle E$, then $\angle C = \angle F$ automatically — no third angle condition is needed. AA is really ‘all three angles equal in disguise.’

Then name the connection to congruence: ‘In congruence, all angles equal didn’t guarantee congruence because we also needed a size constraint. In similarity, all angles equal is sufficient — because similarity doesn’t require same size, only same shape. AAA for congruence fails; AA for similarity succeeds.’

Part C — The epistemological honesty note — say this before notes

‘The shortcuts you found today are called AA similarity, SAS similarity, and SSS similarity. They can each be proved from our axioms. Mathematicians have proved them. We haven’t — and we won’t in this course — because the proofs require techniques we haven’t built yet. What we have is very strong evidence and a deliberately honest choice: we accept these as postulates and use them as tools. Naming the choice is what distinguishes mathematical honesty from just asserting things.’

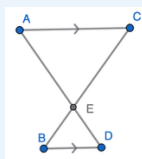
SHORTCUTS REFERENCE — TEACHER COPY

Surface these in the note-making phase. Students should write all three valid shortcuts and understand why each invalid case fails.

Shortcut	What it requires	Why it works
AA~	Two pairs of corresponding angles congruent.	Triangle angle sum forces the third angle: if two angles are fixed, the third is determined. Two angle conditions are sufficient to guarantee all three.
SAS~	Two pairs of corresponding sides in proportion AND the included angle congruent.	The included angle pins the two proportional sides together. The remaining angles and sides are forced. Mirrors SAS congruence exactly.
SSS~	All three pairs of corresponding sides in proportion (same ratio k for all three pairs).	Three sides in proportion completely determine the shape. No angle conditions needed because the angles are forced by the side lengths.
SS (not valid)	Two pairs of sides in proportion, no angle condition.	The angle between the sides is not fixed. Different triangles can share two side ratios without being similar.
SSA~ (not valid)	Two sides in proportion and a non-included angle.	Same ambiguity as SSA in congruence. The configuration is not sufficient.
AAA (not valid for congruence but note for similarity)	All three pairs of angles congruent.	Three equal angles guarantee similarity but NOT congruence. This is AA~ in disguise (the third angle is forced by the first two).

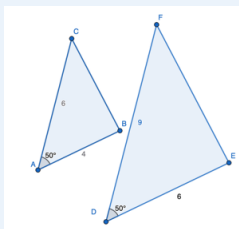
WORKED EXAMPLES — TEACHER REFERENCE

Example 1 — AA~ (overlapping triangles)



Proof: $\angle CEA = \angle BED$ (vertical angles — free from diagram). If $CA \parallel BD$, then $\angle ACE = \angle DBE$ (alternate interior angles). AA~ $\rightarrow \triangle CAE \sim \triangle BDE$. Note correspondence: $C \leftrightarrow B$, $A \leftrightarrow D$, $E \leftrightarrow E$.

Example 2 — SAS~



Proof: $AB/DE = 4/6 = 2/3$. $BC/EF = 6/9 = 2/3$. $\angle ABC = \angle DEF = 50^\circ$ (given). SAS~ $\rightarrow \triangle ABC \sim \triangle DEF$ with $k = 2/3$.

Example 3 — SSS~

Diagram: two triangles with all three sides labeled.

Two triangles. Triangle 1: sides 5, 8, 10. Triangle 2: sides 7.5, 12, 15. Check: $5/7.5 = 8/12 = 10/15 = 2/3$.

Check ratios: $5/7.5 = 2/3$, $8/12 = 2/3$, $10/15 = 2/3$. All three pairs in proportion with $k = 2/3$.
SSS~ $\rightarrow \triangle ABC \sim \triangle DEF$. Correspondence determined by matched side ratios.

STUDENTS MAKE NOTES ON

Students make notes on:

- The six scenarios: which conditions succeeded, which failed, and a counterexample for each failing case
- AA~, SAS~, SSS~: each as a conditional statement, what it requires, why it works (brief)
- Why AA~ is sufficient: triangle angle sum forces the third angle
- Why AAA doesn't guarantee congruence but does guarantee similarity: connect to the size/shape distinction
- The epistemological note: these are accepted as postulates; they can be proved but are not proved here
- One worked example for each shortcut, including the similarity statement with correct correspondence

Narrative Arc — End of consolidation — transition to Lesson 6.4

"We now know when similarity is guaranteed — and we have tools to prove it. The question now is: what can we do with that? Similarity lets us measure things we cannot reach. Next lesson we go outside. You will measure something tall without being allowed to touch it, or use a ruler, or tape measure. The only tool you have is the geometry we've built."

Connection Points

- Draws from Unit 3: the counterexample method and the acceptance of shortcuts as postulates mirrors exactly the Unit 3 approach to SSS, SAS, ASA, AAS
- Draws from Lesson 6.2: the full definition of similarity (six conditions) established in 6.2 is the starting point for asking 'how many do we need?'
- Draws from Unit 1/2: triangle angle sum (Unit 4) explains why AA~ is sufficient — the third angle is forced
- Feeds into Lesson 6.4: AA~ is the shortcut used in every indirect measurement proof (shadow method, mirror method, line-of-sight method)
- Feeds into Lesson 6.5: triangle similarity is the foundation for all proportionality theorems about parallel lines and angle bisectors