Check Your Understanding 8.5

Basic	Intermediate	Advanced
Row 2 Spiral — All circles are similar → *Why proportional reasoning works*		
Basic — Spiral **1.** A circle has radius r; a similar circle has radius $3r$. What is the similarity ratio?	*Intermediate — Spiral* **2.** The smaller circle has a chord of length 8. A corresponding chord in the larger circle subtends the same central angle. Find its length.	*Advanced — Spiral* **3.** Explain why corresponding chords in similar circles scale by the similarity ratio, using the definition of similarity rather than a formula.
Row 3 Spiral — Arc length and sector area → *Applying proportional reasoning*		
Basic — Spiral **4.** A sector has arc length 6π and radius 9. Find the central angle.	*Intermediate — Spiral* **5.** Find that sector's area.	*Advanced — Spiral* **6.** A sector of a different circle has the same arc length 6π but central angle 270°. Find its radius and sector area, then explain how equal arc lengths can give very different sector areas.
Row 4 Spiral — Central and inscribed angles → *The spine of the unit*		
Basic — Spiral **7.** Points A, B, C, D lie on a circle; the inscribed angles ADB and ACB are both 35°. What can you conclude about C and D? Justify with the inscribed angle theorem.	*Intermediate — Spiral* **8.** If the central angle AOB = 94°, find arc AB and the inscribed angle subtending it.	*Advanced — Spiral* **9.** Inscribed quadrilateral ABCD has angle $A = 4x + 10$ and angle $C = 2x + 20$. Find x, then all four angles if angle B = angle D.
Tangent lines and chord theorems → *Consequences of the definition*		
Basic **10.** A tangent line touches a circle at T; the radius OT = 8. (a) What is the angle between OT and the tangent? (b) A point P on the tangent is 6 units from T — find OP. (c) Is P inside or outside the circle? Justify.	*Intermediate* **11.** A chord AB has length 12; its perpendicular bisector passes through center O, and the midpoint M is 5 units from O. (a) Find the radius. (b) Explain why $OM \perp AB$ using the chord-bisector theorem and its proof — not just the statement. **12.** From external point P, two tangent segments touch a circle at T_1 and T_2, with $PT_1 = 4x - 3$ and $PT_2 = 2x + 9$. (a) Find x and each length. (b) Explain why $PT_1 = PT_2$ must always hold, using the radius-tangent theorem and triangle congruence.	*Advanced* **13.** A circle has center O and radius 10. (a) A tangent at T has a point P with OP = 26 — find PT. (b) A chord AB through an interior point Q has AQ = 6 and QB = 12; another chord CD through Q has CQ = 8 — find QD.
Algebra Spiral — Exponent Rules		
Simplify. Write every answer with positive exponents. (a) $(a^4b^{-2})^{-3}$ (b) $(2 \div x^3)^{-2}$ (c) $(5x^0y^2)(2x^3)$		

Answers — CYU 8.5

Row 2 Spiral — All circles are similar

1. 3.

2. $8 \cdot 3 = 24$.

3. A similarity maps the small circle onto the large one, scaling all lengths by 3. A chord's endpoints map to corresponding points, so the distance between them scales by 3 — a direct consequence of the definition of dilation.

Row 3 Spiral — Arc length and sector area

4. $(\theta/360)\cdot 2\pi(9) = 6\pi \rightarrow \theta = 120°$.

5. $(120/360)\cdot\pi(81) = 27\pi$.

6. $(3/4)\cdot 2\pi r = 6\pi \rightarrow r = 4$; area $= (3/4)\pi(16) = 12\pi$. Sector area depends on r^2 as well as the angle, so a small circle with a large angle has far less area than a large circle with a small angle, even at equal arc length.

Row 4 Spiral — Central and inscribed angles

7. Both subtend the same arc AB and are equal, which is exactly what the inscribed angle theorem predicts — C and D are different points on the same arc determined by AB.

8. Arc AB = 94°; inscribed angle = 94/2 = 47°.

9. Opposite angles supplementary: $6x + 30 = 180 \rightarrow x = 25$. A = 110°, C = 70°. B = D = 90°.

Tangent lines and chord theorems

10. (a) 90°. (b) Right angle at T: $OP = \sqrt{(64 + 36)} = 10$. (c) Outside — $OP = 10 > r = 8$.

11. (a) $OA^2 = OM^2 + AM^2 = 25 + 36 = 61$, so $r = \sqrt{61}$. (b) O is equidistant from A and B (both radii), so by the perpendicular bisector theorem O lies on the perpendicular bisector of AB, which meets AB at M at a right angle — hence $OM \perp AB$.

12. (a) $4x - 3 = 2x + 9 \rightarrow x = 6$; each segment = 21. (b) $OT_1 = OT_2 = r$, $\angle OT_1P = \angle OT_2P = 90°$, OP shared $\rightarrow$ $\triangle OT_1P \cong \triangle OT_2P$ (HL), so $PT_1 = PT_2$ by CPCTC.

13. (a) Right angle at T: $PT = \sqrt{(676 - 100)} = 24$. (b) Intersecting chords: $6\cdot 12 = 8\cdot QD \rightarrow QD = 9$.

Algebra Spiral — Exponent Rules

(a) b^6/a^{12}

(b) $x^6/4$

(c) $10x^3y^2$