Lesson 3.5a — Teacher Reference Notes

Proof Progressions

Unit 3: Proof and Congruence • Day 5 of 9

At a Glance	
	Pacing: Ray Allen launch (~8 min) • Progressions 1–3 board work (~45 min) • Task 4 card sort (~15 min) • Consolidation (~12 min) • Notes + CYU begins (~10 min) **Materials:** Boards and markers • Card sort printed, one set per group (Task 4) **Nav instrument rows active:** Rows 1–4 spiral • Row 5 (Proof Structure) and Row 6 (Writing Complete Proofs) activate today **Critical setup:** Only flow chart proofs and paragraph proofs are used in this course. Two-column proofs are not permitted. Enforce this from the first proof attempt. **Groups:** Visibly random throughout — all three progressions are group board work.

OBJECTIVES

By the end of this lesson students can:

- Understand proof as an iterative process — a first attempt is a draft, not a failure.
- Use the backward walk as one planning strategy and recognize that working forward from a given is an equally valid starting point.
- Write a complete flow chart or paragraph proof in which the conclusion of each step feeds into the next.
- Recognize how a flow chart makes the law of syllogism visible.
- Explain why proving triangles congruent often appears as an intermediate step in a proof, even when not explicitly asked.

Why two-column proofs are not used in this course

Two-column proofs allow students to list facts and justifications without constructing a logical argument. The format makes it easy to write individually justified statements that do not necessarily chain together into a genuine deduction. Students found them easier — but that ease was a signal they were not doing the hard mathematical work.

Flow chart proofs and paragraph proofs require students to show the shape of the argument. In a flow chart, boxes and arrows make the law of syllogism visible: you can literally see how the conclusion of one step feeds into the next.

When students ask why they cannot use two-column proofs, suggest they can start there to get the elements they need for their argument, but then they should organize the argument.

LAUNCH — THE RAY ALLEN STORY (≈8 MIN)

I would suggest sharing some form of a story like this

The Ray Allen Story — read aloud or paraphrase

In a 'Letter to My Younger Self' published by The Players' Tribune, NBA Hall of Famer Ray Allen wrote about how much he resented being called a natural shooter. People would watch him drain three-pointers and say he was just gifted. What they didn't see were the thousands of hours — the early mornings, the shooting routine he ran before every game, the same sequence every time, for years. What looked effortless from the outside was the product of more deliberate repetition than most people could imagine.

When a teacher writes a proof on the board and it flows cleanly, it looks the same way. That proof was worked out in advance. The teacher already knows which dead ends to avoid. You are seeing the finished version.

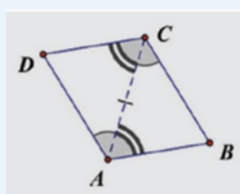
Your first proof attempt will probably be messy. You may start from the wrong place. You may write a step and realize it has no justification. That is not failure — that is what writing a proof actually looks like. The goal today is not a perfect proof on the first try. The goal is to build the habit of reasoning carefully and being willing to revise.

THE PROOF PROGRESSIONS — GROUP BOARD WORK (≈45 MIN)

Students prove that opposite sides of a parallelogram are congruent. The problem is revealed in three stages, moving from heavily scaffolded to completely open. All three stages are board work in visibly random groups.

Progression 1 — Familiar Ground

What is given



In the figure, ∠DAC ≅ ∠BCA and ∠ACD ≅ ∠CAB, and AC is a shared diagonal.
Prove: the opposite sides of the figure are congruent.

The two alternate interior angle pairs are handed to students as given information. The only reasoning required is ASA then CPCTC. Most groups should move through this relatively quickly.

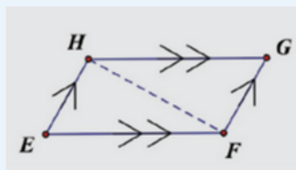
Critical observation to surface after Progression 1

Students proved the two triangles congruent even though the problem only asked for the sides. Name this explicitly: 'You proved the triangles congruent as an intermediate step even though that wasn't what was asked. This pattern will appear constantly. When you want to prove two specific parts are congruent, the path often runs through proving whole triangles congruent first, then using CPCTC.'

Then ask: 'Where did the angle information come from? It was given. But what if it hadn't been given? What would you have needed to know?' This is the bridge to Progression 2.

Progression 2 — One Layer Back

What is given



HGFE is a parallelogram. Diagonal HF is drawn. HG ∥ EF and HE ∥ GF.

Prove: HG ≅ EF.

The angle marks are gone. Students must now justify the alternate interior angles from the parallel sides. This is where groups need the most support.

⚠️ Most common error in Progression 2

Two sets of parallel sides, two pairs of alternate interior angles, each pair coming from a different set of parallel lines using the diagonal as transversal. Students frequently use the wrong parallel lines for a given angle pair.

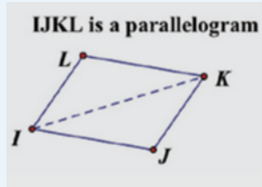
Circulate and ask: 'Which two lines are parallel for that angle pair? What is the transversal? Are those actually alternate interior angles for those two parallel lines?' This forces precision.

The connection to Progression 1

Once groups establish the two angle pairs as alternate interior angles, they have justified exactly what was handed to them in Progression 1. Make this explicit: 'What you just proved in your first step was exactly what was given to you in the first problem. We have now shown where that given came from.'

Progression 3 — The Full Proof

What is given



Given: IJKL is a parallelogram.
Prove: opposite sides of IJKL are congruent.

Only the definition of parallelogram is given. No diagram marks beyond the shape itself. Students must draw the diagonal themselves, use the definition of parallelogram to establish parallel sides, then proceed as in Progression 2.

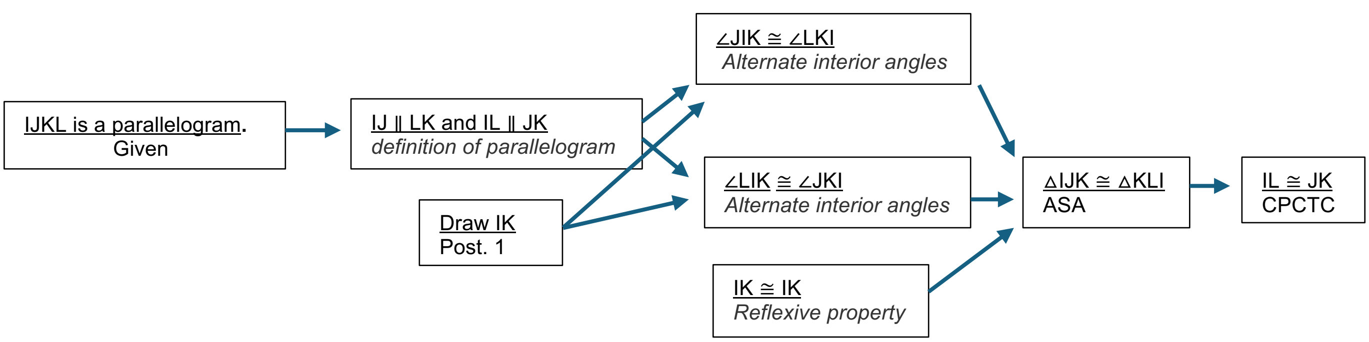
After groups have proofs on the boards — the reveal

'This last one is the complete proof. But look at what we did to get here. In the first problem the angle information was handed to you. In the second you had to justify it from the parallel sides. In the third you had to first establish that the sides were parallel from the definition. Each step was always there — we just revealed it one layer at a time.'

Name the two planning strategies students now have: (1) start from a familiar intermediate step and build in both directions; (2) start from the conclusion and walk backward asking what must have come before. A third approach — writing everything you know from the given and seeing where it leads — is also valid when stuck. Trying a path that does not work is not failure. It is reasoning.

COMPLETE PROOF REFERENCE — OPPOSITE SIDES OF A PARALLELOGRAM

For teacher reference. Use during consolidation to confirm student proofs, not as a model shown before students have worked.



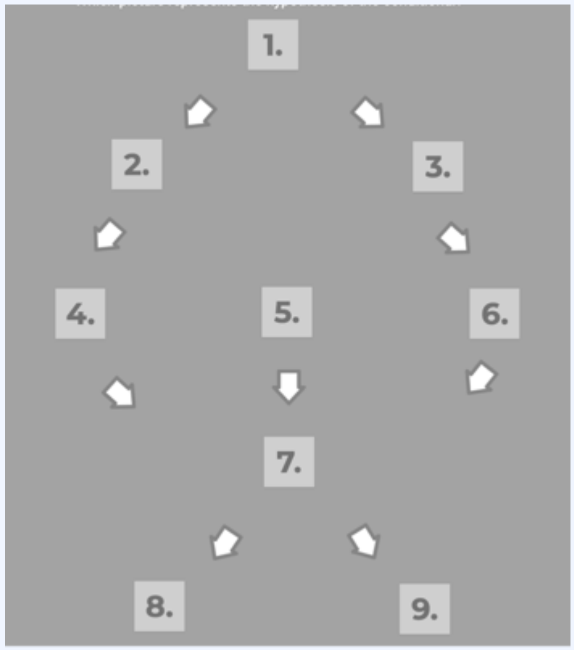
Note

The same proof also establishes IJ ≅ LK by CPCTC from the same triangle congruence. Students who notice this on their own are demonstrating exactly the kind of observation the course is building toward.
Ask about it if it is not volunteered.

TASK 4 — CARD SORT: PROOF WITHOUT WORDS (≈15 MIN)

Card sort: Given: C is the midpoint of AB and C is the midpoint of DE. Prove: DA ≅ EB

Distribute physical card sort (one set per group, printed on card stock). Groups arrange the statement cards into a valid proof matching the following flow (projected on the board)



Purpose and teacher moves

The card sort removes the burden of generating proof statements and focuses entirely on logical ordering. The physical act of moving cards before committing to paper mirrors the iterative drafting process from the Ray Allen story.

As groups arrange cards, ask: 'Why does this card have to come before that one? What would happen to the argument if you swapped them?'

Once groups have arranged their cards, they draw the flow chart on the board with arrows showing logical connections. The card order becomes the proof structure.

This technique — writing what must be true on cards and playing with the flow until it makes sense — is a scaffold you can use going forward with any student who is stuck on where to begin.

CONSOLIDATION (≈12 MIN)

Bring the class together. Cover three things in order.

1. What the three progressions showed

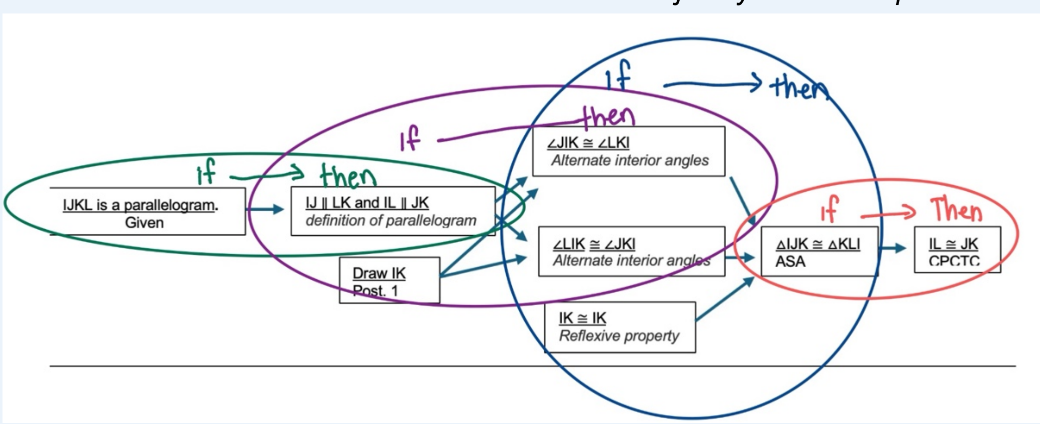
Ask groups to reflect: ‘In Progression 1, where did the angle information come from? In Progression 2, what did you have to do to get it? In Progression 3, where did the whole chain start?’ Guide students to articulate that the three problems were one proof at three levels of scaffolding. The intermediate steps were always there — the scaffolding just hid them.

2. Proving triangles congruent as an intermediate step

Name it explicitly: ‘When you want to prove two specific parts are congruent, the path often runs through proving whole triangles congruent first, then using CPCTC. This is not a detour — it is the argument. You will do this in almost every proof in this unit.’

3. The shape of the argument — law of syllogism made visible

Point to a completed flow chart on the board. Ask: ‘What is each arrow doing? What does it mean for one box to point into another?’ Guide students to articulate: the output of one step is the input of the next — the law of syllogism in geometric form. At some points two boxes both feed into one: both antecedents are needed to justify the consequent.



STUDENTS MAKE NOTES ON

✎ Students make notes on:

- A proof is a chain of conditional statements connected by the law of syllogism. The conclusion of one step becomes the antecedent of the next.
- What can be a starting point (does not need to follow from a previous step): given information, vertical angles, linear pairs, reflexive property, postulates. Everything else must follow from something that came before. All of these still need to be cited.

- Three proof planning strategies: (1) start from a familiar intermediate step; (2) start from the conclusion and walk backward; (3) start from the given and see where it leads.
- Flow chart proof format: statements in boxes, arrows showing logical flow, justifications on or beside arrows. When two statements both feed into one conclusion, two arrows point into the same box.
- Only flow chart or paragraph proofs in this course. No two-column proofs.
- Proving triangles congruent as an intermediate step: even when not asked, proving triangle congruence is often the path to proving specific pairs of sides or angles congruent via CPCTC.

Narrative Arc — End of Lesson 3.5a — close of class

"You just wrote proofs. Real proofs, from definitions you built in Unit 2 and tools you proved in this unit. The first attempt was messy for most groups. That is normal. Ray Allen didn't make every shot the first time either."

"What matters is that you have a process now: start somewhere, follow the logic, revise when a step has no justification. The next lesson you will use this process on problems where the path is not given to you in three stages. That is what proof looks like when it is fully yours."

Connection Points	• Draws from Unit 2: law of syllogism is the structure of a proof chain, made visible in the flow chart format • Draws from Lesson 3.3: free information (vertical angles, linear pairs, reflexive) is used as proof starting points here • Draws from Lesson 3.2: ASA and CPCTC are the main tools in all three progressions • Feeds into Lesson 3.5b: students will present and critique proofs written by peers • Feeds into Lesson 3.5c: scaffolded proof practice with mild/medium/spicy entry points builds on today's methods