Check Your Understanding 3.2

Complete each problem. Write CPCTC in full before using it. Show all work.

Basic	Intermediate	Advanced
Row 1 Spiral — Constructions and What They Prove → *Proof begins with what we can construct from axioms*		
Basic — Spiral Name the postulate or common notion that justifies each claim below. *(a) 'I can draw a circle of any size at any point.'* *(b) 'Two segments that are both equal to a third segment are equal to each other.'*	*Intermediate — Spiral* Explain in one sentence why Euclid's construction of an equilateral triangle on segment AB proves the result, rather than merely showing it.	*Advanced — Spiral* A student wants to copy segment CD onto ray EF using a compass. Describe the construction steps and cite the postulate that guarantees the copied segment has the same length.
Row 2 Spiral — Postulates vs. Propositions → *The foundation: what we accept vs. what we must earn*		
Basic — Spiral Define the following terms in your own words. Give one example of each from Unit 3. *(a) Postulate:* *(b) Proposition:*	*Intermediate — Spiral* In this course, SSS is accepted as a postulate. In Euclid's Elements, SAS is Proposition 4. Explain the difference between treating a result as a postulate versus proving it as a proposition. What are we gaining and giving up with each choice?	*Advanced — Spiral* Why does the order of propositions matter in a deductive system? Give a specific example from Euclid's Elements (or from this unit) where one result must come before another, and explain why reversing the order would break the argument.
Triangle Congruence Shortcuts → *The geometric facts we draw from to build arguments*		
Basic For each pair of triangles, identify which congruence shortcut applies (SSS, SAS, ASA, AAS), or state what additional information would be needed. *(a) △ABC and △DEF: AB = DE, BC = EF, AC = DF.* *(b) △PQR and △XYZ: ∠P = ∠X, PQ = XY, ∠Q = ∠Y.* *(c) △JKL and △MNO: JK = MN, ∠J = ∠M. What additional information is needed for SAS?*	*Intermediate* (a) Draw two different triangles that have two equal side lengths and one equal angle — but the angle is not between the two sides. This disproves SSA. (b) Draw two different triangles that have all three angles equal. This disproves AAA. *(c) Explain in one sentence why SSA fails but ASA does not.*	*Advanced* ABCD is a parallelogram. Diagonal AC is drawn. *(a) Write a congruence statement for the two triangles formed. Use correct correspondence order.* *(b) Which congruence shortcut applies? What information justifies it?* *(c) If you used CPCTC, write its full definition before using the abbreviation.*

Basic	Intermediate	Advanced
Algebra Spiral *Expand*	*Algebra Spiral* Expand each expression completely. (a) $(x + 4)(x - 3)$ (b) $(2a + 5)^2$ (c) $(3y - 1)(2y + 7)$	

CYU 3.2 — Answer Key (Teacher Copy)

Row 1 Spiral

(a) Postulate 3.

(b) Common Notion 1.

Intermediate: The construction operates within the axiomatic system — every step follows necessarily from postulates and common notions, so the conclusion cannot be otherwise. A drawing only shows what appears to be true.

Advanced: Draw circle centered at E with radius CD. The intersection of this circle with ray EF is the endpoint of the copied segment. Postulate 3 guarantees the circle can be drawn; definition of circle (all radii equal) guarantees the copied segment equals CD.

Row 2 Spiral

Basic: Postulate — a statement accepted without proof (e.g., SSS). Proposition — a statement that has been proven from prior results (e.g., the equilateral triangle construction).

Intermediate: A postulate is assumed — we accept it and build on it. A proposition earns its place by being derived from earlier results. Accepting SSS as a postulate saves time but leaves the result unproven; treating it as a proposition gives a deeper justification but requires more foundational work first.

Advanced: If Proposition N depends on Proposition N−1, then N−1 must be established first. Example: in Euclid, Proposition 2 (copying a segment to a given point) relies on Proposition 1 (constructing an equilateral triangle). Reversing the order would use an unproven result.

Congruence Shortcuts

(a) SSS.

(b) ASA.

(c) Need JL = MO (the side between the known angle and a second side — making it SAS).

Int (a) & (b): Accept any valid counterexamples. For SSA: an acute and obtuse triangle can share two sides and a non-included angle. For AAA: two equilateral triangles of different sizes.

Int (c): SSA fails because the non-included angle does not pin the triangle to one shape; ASA works because the included side fixes the gap between two known angles.

Adv (a): $\triangle ABC \cong \triangle CDA$ ($A \leftrightarrow C$, $B \leftrightarrow D$, $C \leftrightarrow A$).

Adv (b): ASA — alternate interior angles ($AB \parallel DC$ and $BC \parallel AD$) plus reflexive side AC.

Adv (c): Corresponding Parts of Congruent Triangles are Congruent.

Algebra Spiral

(a) $x^2 + x - 12$

(b) $4a^2 + 20a + 25$

(c) $6y^2 + 19y - 7$