

Trapezoids and Kites

Unit 5: Quadrilaterals • Day 4 of unit

At a Glance

Pacing: Task 1: Isosceles trapezoid (projected image, all groups) (~30 min) • Task 2: Kite (given individually as groups are ready) (~25 min) • Gallery walk if time allows (~8 min) • Consolidation: both proof sets (~20 min) • Notes recorded at boards throughout

Materials: Boards and markers • Projected image of isosceles trapezoid ABCD for Task 1 • Student note sheet distributed at the start

Nav instrument rows active: Row 4 (Trapezoids and Kites) — all three levels activate today

Notes delivery: Students carry note sheets to the boards from the start. When the teacher approves a group's proof for a property, they record it before moving on. This is different from 5.3's share-and-revise approach — notes are recorded in the moment.

OBJECTIVES

By the end of this lesson students can:

- Prove that an isosceles trapezoid has congruent base angles using the auxiliary altitude construction and HL.
- Prove that an isosceles trapezoid has congruent diagonals using SAS and CPCTC from the base angles result.
- Explain why co-base angles are supplementary without a new proof.
- Prove that a kite has perpendicular diagonals using SSS then SAS.
- Explain why a kite has exactly one pair of congruent opposite angles.
- Identify symmetry as the structural reason behind each proof.

 **Narrative Arc** — Before Task 1 — name what's different about this lesson

"In Lessons 5.2 and 5.3 every shape we studied was a parallelogram. The proofs all used the same move: draw a diagonal, find two triangles, prove them congruent. Today we leave the parallelogram family. Trapezoids and kites do not have two pairs of parallel sides. The hidden triangles are still there — but finding them requires a different auxiliary construction. Watch for where the symmetry lives in each shape. That is where the triangles will be."

TASK 1 — ISOSCELES TRAPEZOID (~30 MIN)

Distribute note sheets before groups begin.

Prompt —

Prompt

ABCD is a trapezoid with $AB \parallel CD$ and legs $AD = BC$. Mark everything you know from the definition. What can you conjecture about the angles? About the diagonals? Test your conjectures. Which can you prove using triangle congruence? What auxiliary lines might help? Write each proven result as a conditional statement. When your teacher checks your proof, record it in your notes before moving on.



⚠ **Three properties to reach — in order**

Property 1 — Base angles congruent: $\angle ADC = \angle BCD$ (lower base) and $\angle DAB = \angle CBA$ (upper base). The proof requires an auxiliary construction: drop perpendiculars from A and B to CD, meeting at E and F. This creates two right triangles with equal hypotenuses ($AD = BC$, given) and equal legs ($AE = BF$, see coaching note below). HL gives congruence, then $\angle ADC = \angle BCD$ by CPCTC.

Property 2 — Diagonals congruent: $AC = BD$. Proof uses Property 1 directly: $\triangle ACD$ and $\triangle BDC$ share base CD, $AD = BC$ (given), $\angle ADC = \angle BCD$ (just proven) $\rightarrow$ SAS $\rightarrow AC = BD$ by CPCTC. Only two steps if Property 1 is treated as a given.

Property 3 — Co-base angles supplementary: $\angle DAB + \angle CDA = 180^\circ$. This follows directly from co-interior angles on $AB \parallel CD$ — no new triangles, no new proof. Name it as a consequence, not a new argument.

★ **The $AE = BF$ step — the step students most often cannot justify**

Both altitudes are perpendicular to CD, so they are parallel to each other (two lines perpendicular to the same line are parallel — corresponding angles are both 90°). Now: $AE \parallel BF$ (just argued), $AB \parallel EF$ (both horizontal, since E and F are on CD). So ABFE is a parallelogram. Since $\angle AEF = 90^\circ$, it is actually a rectangle. In a rectangle, opposite sides are equal, so $AE = BF$.

An alternative argument that also works: AE and BF are both perpendicular to CD , and $AB \parallel CD$, so ABFE is a parallelogram with right angles — a rectangle. $AE = BF$ as opposite sides of the rectangle. Accept either route.

Prompt for groups who are stuck: 'If you drew perpendiculars from A and B to CD, what shape would ABFE form? What does that tell you about AE and BF?'

Hint if groups cannot find the auxiliary lines: 'What if you drew a perpendicular from each top vertex down to the bottom base? What kind of triangles would that create?'

TASK 2 — KITE (≈25 MIN, GIVE INDIVIDUALLY WHEN GROUPS ARE READY)

Draw kite ABCD on each group's board section when you give the task — A at top, C at bottom, B at right, D at left. $AB = AD$ (top pair) and $CB = CD$ (bottom pair). Label the equal sides before they begin.

Prompt — give individually

Prompt

ABCD is a kite with $AB = AD$ and $CB = CD$. Mark everything you know from the definition. Draw both diagonals. What do you notice? Make conjectures. What can you prove about the diagonals? About the angles? The kite has an axis of symmetry. Where is it? How does symmetry help your proofs? When your teacher checks your proof, record it before moving on.

⚠ Two properties to reach

Property 1 — Diagonals perpendicular: $AC \perp BD$. The proof runs in two stages. First: SSS on the large triangles ($\triangle ABC \cong \triangle ADC$, because $AB = AD$, $CB = CD$, $AC = AC$) $\rightarrow \angle BAC = \angle DAC$ (CPCTC — axis AC bisects $\angle A$). Second: SAS on the smaller triangles ($\triangle ABE \cong \triangle ADE$, because $AB = AD$, $\angle BAE = \angle DAE$ from above, $AE = AE$) $\rightarrow \angle AEB = \angle AED$ (CPCTC). Linear pair $\rightarrow$ each 90° . $AC \perp BD$.

Property 2 — One pair of opposite angles congruent: $\angle ABC = \angle ADC$ (the angles at B and D). From $\triangle ABC \cong \triangle ADC$ (established above), $\angle ABC = \angle ADC$ by CPCTC. The other pair ($\angle A$ and $\angle C$) are not generally equal.

Note: a square is technically a kite (two pairs of consecutive equal sides). If this comes up, name it as an observation from the 5.1 bucket sort and move on.

GALLERY WALK (IF TIME ALLOWS — ≈8 MIN)

- 'This group proved the isosceles trapezoid has congruent base angles. Where did the auxiliary lines come from? What made those two triangles congruent?'
- 'The kite proof used SSS on two large triangles first, then SAS on two small ones. Why did they need both steps?'
- 'Both shapes today had a line of symmetry. Where is it in each? How did symmetry appear in the proof — not just as an observation, but as a step that forced two things to be equal?'

CONSOLIDATION — FOUR PROOFS

Work through each proof after the gallery walk. Students should already have their notes from the board work — consolidation confirms and names what they found.



Isosceles Trapezoid — Base Angles Congruent

Given: Isosceles trapezoid ABCD, $AB \parallel CD$, $AD = BC$. Drop perpendiculars from A and B to CD, meeting at E and F.

Statement	Justification
$AD = BC$	<i>Given — isosceles trapezoid definition.</i>
$AE \perp CD$ and $BF \perp CD$	<i>Construction — altitudes from A and B to CD.</i>
$\angle AED = \angle BFC = 90^\circ$	<i>Definition of perpendicular.</i>
$AE = BF$	<i>ABFE is a parallelogram ($AE \parallel BF$ and $AB \parallel EF$). Since $\angle AEF = 90^\circ$, ABFE is a rectangle. Opposite sides of a rectangle are equal.</i>
$\triangle AED \cong \triangle BFC$	<i>HL — hypotenuse $AD = BC$, leg $AE = BF$, right angles at E and F.</i>
$\angle ADE = \angle BCF$	<i>CPCTC.</i>
$\angle ADC = \angle BCD$ ■	<i>$\angle ADE = \angle ADC$ and $\angle BCF = \angle BCD$ (same angles) — lower base angles congruent.</i>

Conditional form: If a trapezoid is isosceles, then the base angles are congruent.



Isosceles Trapezoid — Diagonals Congruent

Given: Isosceles trapezoid ABCD. $\angle ADC = \angle BCD$ (just proven). $AD = BC$ (given).

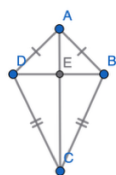
Statement	Justification
$AD = BC$	<i>Given.</i>
$\angle ADC = \angle BCD$	<i>Isosceles trapezoid base angles theorem — just proven.</i>
$CD = CD$	<i>Reflexive property.</i>
$\triangle ACD \cong \triangle BDC$	<i>SAS.</i>
$AC = BD$ ■	<i>CPCTC — diagonals are congruent.</i>

Conditional form: If a trapezoid is isosceles, then the diagonals are congruent.

Property 3 — Co-base angles supplementary (no new proof needed)

Co-interior angles on $AB \parallel CD$ with transversal AD: $\angle DAB + \angle CDA = 180^\circ$. This follows directly from Unit 2/3 transversal work — no new triangle argument needed. State it as a consequence.

Conditional form: If a trapezoid is isosceles, then co-base angles are supplementary.



Kite — Diagonals Perpendicular

Given: Kite $ABCD$ with $AB = AD$ and $CB = CD$. Both diagonals drawn, intersecting at E .

Statement	Justification
$AB = AD$ and $CB = CD$	Given — definition of kite.
$AC = AC$	Reflexive property.
$\triangle ABC \cong \triangle ADC$	SSS — $AB = AD$, $CB = CD$, $AC = AC$.
$\angle BAC = \angle DAC$	CPCTC — diagonal AC bisects $\angle A$.
$AE = AE$	Reflexive property.
$\triangle ABE \cong \triangle ADE$	SAS — $AB = AD$, $\angle BAE = \angle DAE$, $AE = AE$.
$\angle AEB = \angle AED$	CPCTC.
$\angle AEB + \angle AED = 180^\circ$	Linear pair — E lies on BD .
$\angle AEB = \angle AED = 90^\circ$	Congruent angles in a linear pair each equal 90° .
$AC \perp BD$ ■	Definition of perpendicular.

Conditional form: If a quadrilateral is a kite, then the diagonals are perpendicular.

Kite — One pair of opposite angles congruent

From $\triangle ABC \cong \triangle ADC$ (established in the proof above): $\angle ABC = \angle ADC$ by CPCTC. These are the angles at B and D — the vertices where the unequal pairs of sides meet. The other pair of opposite angles ($\angle A$ and $\angle C$) are not generally equal.

Conditional form: If a quadrilateral is a kite, then one pair of opposite angles is congruent. The congruent pair is at the vertices between the unequal sides.

STUDENTS MAKE NOTES ON

Students make notes on:

- Isosceles trapezoid — Property 1 (base angles): conditional form, auxiliary construction described (altitudes from top vertices), HL proof summary
- Isosceles trapezoid — Property 2 (diagonals equal): conditional form, SAS proof summary citing Property 1
- Isosceles trapezoid — Property 3 (co-base angles supplementary): conditional form, why this follows without a new proof
- Kite — Property 1 (diagonals perpendicular): conditional form, two-stage SSS→SAS proof summary
- Kite — Property 2 (one pair opposite angles equal): conditional form, which pair and why not the other
- Symmetry as proof strategy: written in own words (from note sheet prompt)

 Narrative Arc — End of consolidation — transition to Lesson 5.5

“We have now proven properties for every major quadrilateral type: parallelograms, rectangles, rhombi, squares, isosceles trapezoids, and kites. All of it from the same toolkit: find the hidden triangles, prove them congruent, extract by CPCTC. Next lesson we ask: can we prove the same things a different way? Using only coordinates, slope, and distance? And if both approaches work — what does that tell us?”

Connection Points

- Draws from Unit 3: HL is introduced as a special case of AAS for right triangles; SSS and SAS are used in all four proofs
- Draws from Unit 2/3: co-interior angles (Property 3 of isosceles trapezoid) require no new argument
- Draws from Lesson 4.3: the base angles theorem is not directly used here, but the auxiliary altitude construction mirrors the altitude proof from Lesson 4.2
- Feeds into Lesson 5.5: perpendicular diagonals (kite) and equal diagonals (isosceles trapezoid) appear as coordinate classification checks
- Feeds into Lesson 5.6 Advanced (Q16): proving $\triangle ABE$ is isosceles requires chaining both isosceles trapezoid results