Unit 1

Foundations, Coordinates, and the First Acts of Proof

Unit Navigation Instrument

Big Idea: Shared language and measurement are the foundation geometry builds on — precise definitions make precise reasoning possible.

Subtopic	Basic	Intermediate	Advanced
What makes a good definition *→ Building tools for shared language*	Can name both elements of a good definition: classification and differentiation.	Can construct a definition from a set of examples and non-examples.	Can write a novel definition (e.g., for a bicycle or a child) that correctly includes and excludes the right things.
The foundational objects *→ Naming what we reason about*	Can define point, line, ray, segment, and plane; can define collinear, coplanar, parallel, and perpendicular.	Can identify each object in a diagram and name it using correct notation.	Can explain why undefined terms are necessary, and why two points are insufficient to define a plane.
Distance and midpoint in the plane *→ Measuring what we reason about*	Can calculate the distance between two points and find the midpoint of a segment.	Can explain how the distance formula is the Pythagorean theorem in disguise; can find a missing endpoint given a midpoint.	Can apply distance using scale factors; can connect the taxi-cab and Euclidean formulas geometrically.
Polygons and area in the plane *→ Applying measurement to figures*	Can classify a polygon by number of sides and concavity; can find the area of rectangles and triangles in the coordinate plane.	Can find the perimeter of a polygon with slanted sides using the distance formula; can find the area of a compound figure.	Can derive the area formula for a triangle from the rectangle formula; can explain why slanted-side perimeter requires the distance formula rather than counting grid squares.
Angle relationships *→ The first deductive results*	Can measure and classify angles; can identify and define linear pairs, vertical angles, supplementary, complementary, and adjacent angles.	Can use angle relationships to solve for unknown values algebraically.	Can construct a convincing argument that vertical angles must be congruent — the first proof of the course.

Use this instrument regularly — It is embedded in every CYU set and on the differentiation day. The goal is not a one-time self-assessment at the end of the unit. It is a living record of where your students understanding is growing and where it still needs work. Return to it often.

A note on how the downloadable navigation instruments work: the version above shows descriptors only, which is appropriate for the unit map. All student-facing downloadable versions have specific problems or problem numbers in each cell, aligned to the Check Your Understanding problems and differentiation day practice. Students look at the problem they just tried, find the row it belongs to, and mark where they stand. Each new lesson adds new rows to the instrument, but also cycles back through rows from previous lessons — giving students repeated practice on earlier material and building the spaced repetition that makes learning stick.

A survey note worth sharing honestly: some students found the verbal descriptors in the boxes confusing on their own, particularly early in the year before they understood what the rows meant. They are still worth keeping. Students who encounter the same row descriptor paired with a specific problem type repeatedly — across homework sets, CYU problems, and differentiation days — gradually develop a genuine understanding of what the language means and how the ideas connect. The descriptors also build mathematical vocabulary over time. But teachers should not assume that students will understand the rows immediately from the words alone. The pairing with problems is what makes them meaningful.

A Note on This Unit

This unit will look familiar to anyone who has taught or taken a traditional geometry course. Definitions, notation, distance and midpoint, area and perimeter, angle relationships. In many ways it mirrors the opening chapter of most geometry textbooks.

That is not an accident, and it is worth understanding why. Euclid's Elements begins not with theorems but with twenty-three definitions before a single axiom is introduced. Geometry, as a deductive system, requires shared language before it can support shared reasoning. Beginning with vocabulary and basic relationships is not a modern concession to curriculum standards — it is ancient.

Coordinate geometry, introduced by René Descartes in La Géométrie in 1637, did not exist until about two thousand years after Elements was written. From a purely philosophical standpoint, blending Euclidean geometry with Cartesian methods at the very start of a course is an anachronism. If I were designing this course free of all constraints, I might begin with logic, agree on axioms, and only then deduce geometric facts. That approach is philosophically elegant. It is also, in my experience, pedagogically unwise.

After four days of Unit Zero, even four very good days, we do not yet have a fully developed thinking classroom. We have momentum, norms in formation, and students who are beginning to trust that their ideas matter. What we do not yet have is durability. This unit is designed to continue building that durability while grounding students in content they will need repeatedly throughout the year.

There is also something more happening here than vocabulary review. By the end of this unit, students will have made their first deductive argument. Before receiving any formal instruction in proof writing, the majority of my students produced a convincing argument that vertical angles must be congruent — directly from the definitions they had built themselves. That was not accidental. It was a continuation of the epistemological work begun in Unit Zero: conjecture demands justification.

A Note on Notation

One of the clearest lessons from assessing this unit was that notation was under-reinforced in lessons and showed up as a significant gap on the test. Students who could correctly define a ray could not reliably write the symbol for one. Students who understood parallel lines could not express that relationship symbolically.

Notation is not a decoration. In a deductive system, symbolic precision is what allows reasoning to be shared and verified. A student who understands the concept but cannot write it unambiguously is a student whose understanding cannot yet be communicated — which, in a proof, amounts to the same thing as not understanding it.

The revised lesson sequence addresses this more deliberately. The card task in Lesson 1.1 Day 2 is specifically designed to build notation through active use rather than passive copying. Make it a genuine consolidation priority, not a fun warm-up activity.

On Assessment and Quizzes

In the first year teaching this unit, daily quizzes were used to generate feedback and grades. They produced useful data — but they consistently crowded out consolidation time, and the culture cost was real. The lesson sequence below is written without daily quizzes.

Lesson Sequence Overview

Lesson 1.1, Day 1: What is a definition? (shoe task, circular definition moment, undefined terms)

Lesson 1.1, Day 2: Notation and the foundational objects (card task, finish definitions, first use of navigation instrument)

Lesson 1.2: Distance and midpoint (taxicab geometry, distance formula, midpoint, proportional reasoning close)

Lesson 1.3: Polygons and area (security camera task, polygon definitions, triangle-in-rectangle)

Differentiation Day: Cumulative practice, navigation instrument, spaced retrieval

Lesson 1.4: Angle relationships (naming task, interior/exterior, linear pairs and vertical angles, first proof)

Optional review day: Navigation instrument, targeted practice based on diagnosed gaps