

Similarity in Light and Shadow

Unit 6: Similarity • Day 1 of unit

At a Glance

Pacing: Task 1: Mount Rushmore launch (~20 min) • Task 2 Parts A–C: shadow investigation (~40 min) • Task 2 Part D: equilateral triangle challenge (~10 min) • Consolidation: what stayed the same? (~15 min) •

Materials per group: Pre-cut scalene triangle (cardboard or stiff paper — stencil in downloads) • Flashlight • Meter stick or measuring tape • String • Protractor • Masking tape (for marking shadows on floor) • Student worksheet (downloads) • Equilateral triangle (for Part D)

Nav instrument rows active: Row 1 (Similarity and Scale Factor) begins activating today — Basic level only. No formal definition yet.

Prepare before class: Print and cut scalene triangle stencils. Test flashlights.

Do not name: similarity, scale factor, or dilation during Tasks 1 or 2. These words belong to the consolidation and to Lesson 6.2.

Source credit: Task 2 is adapted from Matthew Engle’s 2017 Rosenthal Prize lesson ‘Bringing Similarity Into Light’ (MoMath). <https://momath.org/wp-content/uploads/2019/03/2017-Rosenthal-Prize-Bringing-Similarity-Into-Light-Matthew-Engle-2019-03-22.pdf>

OBJECTIVES

By the end of this lesson students can:

- Explain why proportional reasoning works to estimate real-world dimensions from a photograph.
- Identify and articulate what stays the same (angles, ratios of corresponding sides) and what changes (actual lengths) when a shape is scaled from a point.
- Explain that scaling from a point means each vertex lies on a straight line through the center of dilation.
- Recognize that orientation does not disrupt scaling — the ratios and angles are preserved even when the flashlight is at an angle, give the object and its projection plane are parallel.
- Articulate informally why the scalene and equilateral triangles can produce the same shadow without being the same shape.

Narrative Arc — Open class — before Task 1

“This unit starts the same way every unit in this course has started: with a question you can already make progress on. You don’t need a definition. You don’t need a formula. You need to notice something, test it, and ask why it works.”

“The question for the next two days: what does it really mean for two shapes to be the same shape, even when they are different sizes? Don’t answer it yet. Just hold it.”

TASK 1 — MOUNT RUSHMORE (≈20 MIN)

Read aloud, then project the Mount Rushmore image

Launch — say exactly this

'A tour guide once told me that the sculpture of George Washington's head is about 60 feet tall. I've always wondered: how big do you think his nose is?'



Do not provide any further instructions. Do not say ratio, proportion, scale, or similar.

Task for groups

Your job is to make a reasonable estimate for the size of Washington's nose. You may measure the image. Be prepared to explain why your method works — not just what answer you get.

⚠ What to watch for

Definitions: Continuing a theme from throughout the class you must agree on your definition of the nose is, where does it start and end. Same for the head. These things feel like the definition should be assumed, but they cannot be.

What students will do: measure head height and nose length in the image, compute nose/head ratio, multiply by 60 feet. Most groups land on approximately 20 feet. Some estimated the nose was $\frac{1}{3}$ of the face.

Circulation questions — use these to preserve openness and surface assumptions:

- 'Why does that ratio make sense?'
- 'How do you know those two measurements correspond?'

- 'What assumptions are you making about the photograph? What if it were taken at an angle?'
- 'Would your method still work if the image were larger or smaller?'

Extension prompts for early finishers (give one at a time):

- 'If someone made a statue of you that was 80 feet tall, how big would your nose be?'
- 'If I only told you the nose size, could you estimate the head size? Try it with Lincoln.'
- 'Two groups used correct reasoning but got different answers. How is that possible?'
- 'Would this method work on a selfie? Why or why not?'

Whole-class synthesis — before releasing Task 2

Ask: 'Why did this work? What stayed the same from the picture to the sculpture? What changed? What would make an estimate unreasonable?'

Record student language informally on the board — do not correct or formalize it yet:

- 'the ratios stayed the same'
- 'everything scaled evenly'
- 'the shape didn't change'
- "the camera was parallel to the plane of the face"

Leave this language on the board. Students will connect it to Task 2's findings in the consolidation.

TASK 2 — SIMILARITY IN LIGHT AND SHADOW (≈50 MIN)

Distribute materials. Groups work at their floor workspace. Give all groups Parts A–C simultaneously. Give Part D individually when groups have completed A–C.

Part A — Parallel Triangle, Different Heights (≈15 min)

Students hold their scalene triangle parallel to the floor and shine the flashlight straight down on it. They trace the shadow with masking tape, then measure and record.

What students measure

T1: length of each side of the cardboard triangle.

T2: length of the matching side of the shadow.

T2/T1: ratio for each side. Are all three ratios equal?

L1: distance from light to each vertex of the cardboard triangle.

L2: distance from light to the matching shadow vertex.

L2/L1: ratio for each vertex. Are all three ratios equal? Does $L2/L1 = T2/T1$?

Key noticing goal — confirm before groups move on

All three side ratios $T2/T1$ are approximately equal. All three vertex-distance ratios $L2/L1$ are approximately equal. And $T2/T1 \approx L2/L1$. The scale factor is the same everywhere. Ask: 'Are all your ratios the same? Why do you think that is?'

Part B — Different Heights, Same Shadow (≈10 min)

Students try to reproduce the same shadow using a different combination of flashlight height and triangle height. They measure $L1$ and $L2$ again for the new configuration.

What this demonstrates

As long as $L2/L1$ is the same, the shadow is the same size. Students discover that the scale factor depends only on the ratio of distances from the light source — not on the absolute heights.

If a group finds this hard to believe: ask them to compute $L2/L1$ for both configurations and compare to $T2/T1$.

Part C — Flashlight at an Angle (≈10 min)

Students hold the triangle parallel to the floor but shine the flashlight from an angle (still from a fixed point). They measure side ratios and vertex-distance ratios again.

Key noticing goal

Even with the flashlight at an angle, $L2/L1$ still equals $T2/T1$. Orientation of the light does not change the scale factor if the object and its projection are on parallel planes. Students measure angles and confirm that corresponding angles in the shadow match the original.

This is the key conceptual shift from Part A: it is not the direction of the light that determines the scale factor, but the ratio of distances from the light to the object and shadow. Scaling happens from a point.

Part D — The Equilateral Triangle Challenge (≈10 min, give individually)

On the floor is an equilateral triangle outline taped in advance. Students are given an equilateral triangle (same size or different) and challenged:

Challenge

'Can you position the flashlight and your triangle so that the shadow matches the taped triangle exactly?'

There is no single method. Productive struggle is expected. The point is the shadow of an equilateral triangle can be made to match any scalene triangle if it is not held parallel to the floor.

★ **The deeper question** — pose this after groups succeed or spend 5 minutes

Note: the scalene triangle and the equilateral triangle are not the same shape. But both can cast the same shadows. What does that tell us?

WORKSHEET REFERENCE — TEACHER COPY

The measurement tables below match the student worksheet. Use during Task 2 to confirm group measurements and ratios.

Trial 1 — Side Lengths

	T1 (original)	T2 (shadow)	T2 / T1
Side 1	_____ cm	_____ cm	_____
Side 2	_____ cm	_____ cm	_____
Side 3	_____ cm	_____ cm	_____

Trial 1 — Vertex Distances

	L1 (light to triangle)	L2 (light to shadow)	L2 / L1
Vertex 1	_____ cm	_____ cm	_____
Vertex 2	_____ cm	_____ cm	_____
Vertex 3	_____ cm	_____ cm	_____

Trial 2 — Vertex Distances (different heights, same shadow)

	L1 (light to triangle)	L2 (light to shadow)	L2 / L1
Vertex 1	_____ cm	_____ cm	_____
Vertex 2	_____ cm	_____ cm	_____
Vertex 3	_____ cm	_____ cm	_____

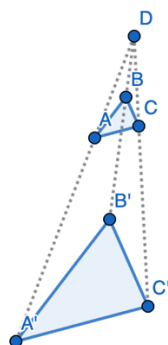
Trial 3 — Vertex Distances (light at an angle, triangle still parallel to floor)

	L1 (light to triangle)	L2 (light to shadow)	L2 / L1
Vertex 1	_____ cm	_____ cm	_____
Vertex 2	_____ cm	_____ cm	_____
Vertex 3	_____ cm	_____ cm	_____

CONSOLIDATION — WHAT STAYED THE SAME? (≈15 MIN)

Bring the class together. Draw on the board:

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Consolidation questions — ask in this order

‘What stayed the same in every experiment?’

Draw out: angles (all corresponding angles equal), ratios of corresponding sides (all equal to the same scale factor), the fact that each vertex lies on a straight line from the light.

‘What changed?’

Draw out: actual side lengths, actual distances.

‘How did you know which sides corresponded?’

This is the correspondence question — the most important habit of the unit. Students should say: the sides that connect the same pair of vertices in the original and the shadow. Draw the matching sides on your board diagram.

‘Every vertex lies on a straight line from the light. What shape does that create?’

Students will say: a pyramid. Confirm: yes — the flashlight, the triangle, and the shadow form a triangular pyramid. Every horizontal cross-section parallel to the base is a similar triangle.

The conceptual payoff — say this

‘This is why similarity shows up everywhere in 3D problems. When things scale from a point, cross-sections stay similar. The flashlight shows you the engine behind that idea: a center, rays extending from it, and the shape at every distance a scaled version of the shape at every other distance.’

‘Tomorrow, we formalize what you’ve discovered. Today’s job was to convince yourself it’s true.’

Brief Cavalieri’s Principle mention — for curiosity, not mastery

If a group or the whole class seems ready for it: ‘The idea that every cross-section of this pyramid is a similar triangle is connected to something called Cavalieri’s Principle — a method for comparing volumes and areas by looking at cross-sections. You’ll see it again

in calculus. For now, just notice: similarity is not just about flat figures. It lives inside 3D shapes.'

 **Narrative Arc** — End of consolidation — transition to Lesson 6.2

"You've seen two things today: proportional reasoning works to scale from a photograph, and scaling from a point in space preserves ratios and angles. Tomorrow we ask: can we make that precise? What exactly does it mean for two figures to be similar — not just to look similar, but to be rigorously the same shape at different sizes? And how little information do we need to establish that for triangles?"

Connection Points

- Draws from Unit 1: proportional reasoning (ratios of corresponding lengths) is the engine of Task 1; the language of correspondence is directly from Unit 3 (CPCTC)
- Draws from Unit 3: the distinction between drawing and proof — today is the experiential phase. Lesson 6.2 formalizes it.
- Feeds into Lesson 6.2: everything students name today as 'what stays the same' becomes the definition of similarity tomorrow
- Feeds into Lesson 6.3: the shadow experiment demonstrates dilation from a point; AA, SAS~, SSS~ are the triangle similarity shortcuts that follow
- Feeds into Lessons 6.5–6.6 (scaling laws): the pyramid cross-section observation foreshadows why area scales as the square and volume as the cube of the scale factor