Lesson 7.3 — Teacher Reference Notes

Choosing the Steepest Safe Path

Unit 7: Trigonometry • Days 4–5 • Day 1: Avalanche Decision + Naming Inverse Trig • Day 2: ADA Ramp + Mixed Practice

At a Glance	
	Day 1 pacing: Avalanche story + task (~35 min) • Consolidation: naming inverse trig (~20 min) • Guided notes + CYU start (35 min) **Day 2 pacing:** ADA ramp investigation (~20 min) • Mixed practice at boards (~60 min) • Discernment discussion + CYU (~10 min) **Materials:** Calculators • Class trig table (still posted) • Measuring tape for ADA ramp • Boards and markers **Nav instrument rows active:** Row 3 (Solving for Missing Angles) activates today — all three levels **Year 1 key insight:** Students must identify the 45° case (Option A) first using the isosceles right triangle argument, not inverse trig. If they struggle, redirect: ‘Which of these can you figure out without a calculator or the table?’

OBJECTIVES

By the end of this lesson students can:

- Use inverse trig functions to find an unknown angle from a known ratio.
- Distinguish between when to use regular trig (angle known → find side) and inverse trig (sides known → find angle).
- Read the class trig table in reverse as estimation before computing with the calculator.
- Identify which inverse function to use given which sides are known.
- Explain why $\sin^{-1}$ is a function-inverse, not a reciprocal.

The conceptual shift — name this before Day 1

In Lessons 7.1 and 7.2, students moved from a known angle to an unknown side. Today they reverse: from known sides to an unknown angle. Inverse trig functions are not a new idea — they are the same functions run backwards. The class trig table already contains everything needed: reading right-to-left instead of left-to-right is inverse trig.

DAY 1 — THE AVALANCHE DECISION (≈35 MIN)

Task

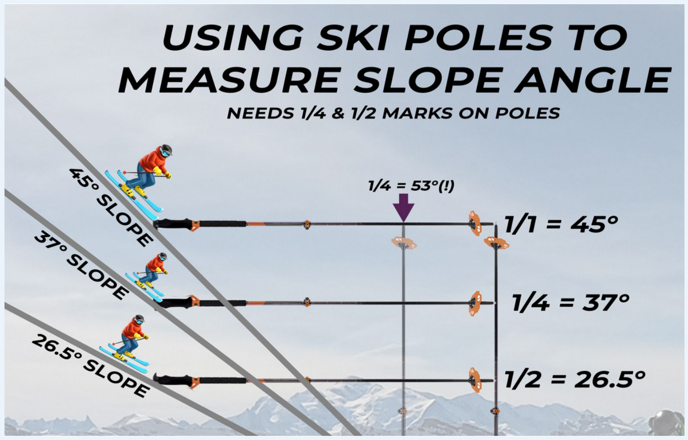
“You and your friends are backcountry skiing. A storm rolled in faster than expected. Visibility is dropping. You need to descend quickly, but avalanche safety training warns that slopes between 30° and 45° are the most dangerous. You don’t have your clinometer. But you remember a trick: place one ski pole vertically, hold another perpendicular to it until it touches the mountainside, and you can estimate the rise over run as a fraction of one pole’s length. You test five possible routes. Which route is the steepest that is still safe?”

Board prompt

Option A: rise:run = 1:1   Option B: 3/4:1   Option C: 1/2:1   Option D: 2/5:1
Option E: 3/5:1

Before using calculators: use the class trig table to estimate which routes are safe. What angles do these ratios correspond to? Then confirm with guess-and-check on calculators. Which route is the steepest safe option?

Here is an image from alpine tutorials you may find helpful:



⚠️ **The 45° case should come first**

Option A (1:1) should be the first one any group solves — and they should solve it without inverse trig. A 1:1 rise:run is an isosceles right triangle, so the angle is 45°. If a group reaches for the calculator immediately, redirect: 'Which of these can you figure out without any trig at all?'

Year 1 moment worth naming: one student observed 'Anytime the tangent is greater than 1 the angle would be greater than 45° and thus in the safe range.' This is excellent mathematical thinking. Name it explicitly when it happens and ask: why must that be true?

Answer key — teacher reference:

Option	Ratio	Angle	Danger?	
A	1.0	45°	YES — at limit	
B	0.75	36.87°	YES — in range	
C	0.50	26.57°	Safe	Steepest safe below 30°
D	0.40	21.80°	Safe	
E	0.60	30.96°	YES — in range	

The guess-and-check inefficiency is intentional

For Options B–E, students must use the class table to estimate, then guess-and-check on the calculator (entering trial angles, checking the tangent, adjusting). This inefficiency is pedagogically essential: it makes the inverse trig function feel necessary rather than arbitrary. Name this in consolidation.

DAY 1 CONSOLIDATION — NAMING THE INVERSE FUNCTIONS (≈20 MIN)

After debriefing the activity, transition to guided notes. Open with:

Say this before the notes

'That was inefficient. You were essentially solving $\tan\theta$ = known ratio for θ, and you had to guess-and-check because you had no direct tool. There is a direct tool. It's called the inverse tangent function.'

The notation warning — say this explicitly

In algebra, x^{-1} means 1/x (reciprocal). But $\sin^{-1}$ is the inverse function, not the reciprocal. $\sin^{-1}(x)$ means 'the angle whose sine is x' — not 1/sin(x). The notation borrows the function-inverse symbol f^{-1}, not the exponent notation. These are completely different mathematical ideas that happen to share a symbol.

Also introduce the arcsin / arccos / arctan names. These are used interchangeably with the $^{-1}$ notation and are often clearer precisely because they avoid the reciprocal confusion.

The history of tables — worth one minute

For centuries, finding an angle from a ratio meant consulting a table — exactly what students did today. Teams of human 'computers' spent careers calculating and verifying trigonometric tables for astronomy, navigation, and surveying. A 1° error in a table could misplace a ship or corrupt an astronomical prediction. Frustration with human calculation errors was a direct motivation for early mechanical computing — including Babbage's Difference Engine in the 19th century. When students press $\tan^{-1}$ today, they are using algorithms refined over two millennia.

STUDENTS MAKE NOTES ON

✎ **Students make notes on:**

- The functional distinction: regular trig (angle → ratio) vs. inverse trig (ratio → angle)
- The three inverse functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ — also written arcsin, arccos, arctan
- The notation warning: $\sin^{-1}$ is not 1/sin. It is the inverse function.
- Calculator use: which key to press, what the input must be (a ratio between 0 and 1 for $\sin^{-1}/\cos^{-1}$, any positive number for $\tan^{-1}$)
- One worked example from the avalanche task

DAY 2 — ADA RAMP INVESTIGATION (≈20 MIN)

Prompt — give to all groups

"According to the Americans with Disabilities Act (ADA), all public ramps must not exceed an incline angle of approximately 4.76°. We have six ramps in this school. Each group will be assigned a ramp — some will work from the top and some from the bottom. Using nothing but a ruler, a calculator, and your own cleverness, your group will determine if your ramp is in compliance with the ADA."

Year 1 moment — the arcsin insight

One group was measuring the ramp and realized they had measured the hypotenuse (along the ramp surface) instead of the horizontal run. Two students were frustrated, thinking they had to redo everything. A third student said: 'Can't we just use arcsin instead of arctan?' The others agreed after she explained. They were also proud to discover the ramp was not in compliance.

This moment is worth naming in class if the situation arises. It shows students that they have real flexibility — they can choose which ratio fits what they measured.

DAY 2 — MIXED PRACTICE AT THE BOARDS (≈50 MIN)

Post problems one at a time. Students must identify which tool the problem requires before solving. The discernment question is the most important part of each problem.

The discernment question — ask before every problem

'Do you know an angle or are you trying to find one? What information do you have? Which tool does that call for — regular trig or inverse trig?'

Pythagorean triples for mixed practice — teacher reference:

Short leg	Long leg	Hypotenuse	Smaller θ	Larger θ	Good problem prompts
3	4	5	36.87°	53.13°	Find the smaller angle ($\sin^{-1}$ or $\tan^{-1}$). Verify with complementary angle sum.
5	12	13	22.62°	67.38°	Give hyp and one leg → find angle. Then use result to find the other leg with regular trig.
8	15	17	28.07°	61.93°	Give both legs → find both angles. Check: do they sum to 90°?
7	24	25	16.26°	73.74°	Give one angle and one leg → find hyp and other leg (regular trig).
20	21	29	43.60°	46.40°	Near 45° — interesting because the two angles are close. Good for noticing how the ratios change.

How to use this table

Each row is a problem set: give students some sides and angles, withhold others. Mix regular trig problems (give angle and one side, find the other) with inverse trig problems (give two sides, find an angle). The 'Good problem prompts' column suggests one strong problem per triple.

CYU 7.3 — ANSWER KEY

Q	Answer	Teacher note
1	$\theta = \sin^{-1}(7/25) \approx 16.26°$. Adjacent = $\sqrt{625-49} = 24$. Verify: $\cos^{-1}(24/25) \approx 16.26°$. ✓	This is the 7-24-25 triple. The verification step is key — both routes give the same angle.
2	$\theta = \tan^{-1}(0.5/6) = \tan^{-1}(0.0833) \approx 4.76°$.	This is the ADA maximum. Connects to the ramp investigation.
3	$\theta = \cos^{-1}(4/10) = \cos^{-1}(0.4) \approx 66.4°$.	Ladder angle is with the ground. Students sometimes confuse the angle with the wall (23.6°). Ask: which angle is being requested?
4	$\theta = \tan^{-1}(8/100) \approx 4.57°$. Well below 30° — very shallow.	A road gradient of 8% (common real-world notation) = 8m/100m.
5	Table estimate: between 30° ($\sin=0.5$) and 45° ($\sin\approx0.707$), closer to 37°. Exact: $\sin^{-1}(0.6) \approx 36.87°$. adj = 0.8, $\cos\theta = 0.8$, $\tan\theta = 0.75$.	This is the 3-4-5 triangle scaled. Push the table estimate — is their estimate reasonable?
6	$\sin^{-1}$ takes a ratio (input) and returns the angle that produces it (output). Works reliably for 0°–90° in this unit because our trig definition is based on right triangles. Each acute angle maps to exactly one ratio, so the inverse is well-defined.	Best answers explicitly connect invariance: because $\sin\theta$ is the same for every right triangle with angle θ, knowing the ratio determines the angle uniquely.
7	The hypotenuse is always the longest side, so opposite < hypotenuse, meaning $\sin\theta < 1$ always. $\sin^{-1}(1.2)$ asks for an angle whose sine exceeds 1 — geometrically impossible. The calculator correctly reports no real solution.	This is a deep question. The constraint is geometric, not computational.

Narrative Arc — End of consolidation — transition to Lesson 7.4

"You now have the full toolkit: find a side from an angle, find an angle from sides. Next lesson we ask: are there right triangles so special that we never need a calculator for them at all?"

Connection Points	• Draws from Lesson 7.1: the class trig table is now read in reverse — the same table, the same values, opposite direction • Draws from Lesson 7.2: the percent-error question from 7.2 Day 2 planted the seed that angle-to-value and value-to-angle are different operations • Draws from Unit 6: arctan(1/12) for the ADA ramp connects to the slope/angle relationship from indirect measurement • Feeds into Lesson 7.4: the 45° case solved without a calculator in this lesson foreshadows the 45°-45°-90° special triangle • Feeds into Lesson 7.5: the discernment habit (which tool?) is the metacognitive skill practiced in the mixed problem set