

Organizing and Generalizing

Unit 0: Problem Solving and Mathematical Habits • Day 2 of 4

At a Glance

Pacing: ~45 min tasks + gallery walk • ~45 min consolidation + culture activities

Materials: Boards and markers

Mathematical habits introduced today: Organizing — structure reveals pattern • Generalizing — moving to variables shows something is always true

Jo Boaler video: youtu.be/ZUHs_xR7I9M — 8 minutes. Watch before class. Plan to show it during the culture portion of the lesson.

OBJECTIVES

By the end of this lesson students:

- Have used organizing as a strategy to reveal pattern in a problem.
- Have seen how introducing variables is the key step from ‘this works for these cases’ to ‘this always works.’
- Have co-created a group work rubric and begun using it for self-assessment.

THINKING TASK

Palindromes (≈35 min)

Source: Thinking Mathematically — Mason & Stacey

Prompt

A number like 12321 is called a palindrome because it reads the same backwards and forwards. A friend claims that all 4-digit palindromes are divisible by 11. Are they?

Support with process questions only:

- What is the smallest 4-digit palindrome?
- How can you organize the ones you’ve tried?
- Do you see a pattern in how palindromes increase?
- Can you find a counterexample?
- Can you generalize the pattern with variables?

The mathematical resolution

Every 4-digit palindrome has the form ABBA. Using place values: $1000A + 100B + 10B + 1A = 1001A + 110B = 11(91A + 10B)$. Therefore every 4-digit palindrome is divisible by 11 — no exceptions.

Key move to name: introducing variables (A and B for digits) is what allowed us to prove this for every palindrome at once, not just the ones we tested. That is generalizing.

Extension ladder: Does this work for 5-digit palindromes? 6-digit? What is the pattern? Can they generalize?

CURATED GALLERY WALK (~8 MIN)

Look for boards during the task. Prioritize:

- A board that organized palindromes in increasing order — this makes the +110 pattern visible.
- A board that introduced variables — let this group present first in the walk.
- Any board that attempted a counterexample seriously — celebrate the attempt regardless of outcome.

Use the I Notice / I Wonder protocol during the gallery walk. Ask students: ‘What do you notice about this board? What do you wonder?’ This builds a culture where incomplete work is worth examining.

CONSOLIDATION — TWO PARTS

Part 1: Consolidate the task. Walk through the algebraic generalization with the class. The consolidation point is the process: specialize → organize → generalize. Students do not need to take notes on the palindrome result itself — the lesson is the procedure.

Part 2: Group work rubric. Ask groups to list behaviors they have seen in good group work and bad group work. Use the contrast format: one column of positive behaviors, one of their antitheses. After groups share, photograph the combined list. Use it to create a formatted rubric to post at workstations for the rest of the year.

Group work rubric — starting framework (Liljedahl)

Positive: Open to others’ ideas • Respectful • Actively inclusive • Sharing the marker • Encouraging

Negative antitheses: Closed to others’ ideas • Disrespectful • Actively excluding • Hogging the marker • Discouraging

Use students’ own language wherever possible. For deeper guidance on building and using this rubric, see Chapter 12 of Building Thinking Classrooms in Mathematics by Peter Liljedahl.

Culture note — Jo Boaler video and class norms

Show the Jo Boaler video on growth mindset and learning math (youtu.be/ZUHs_xR7l9M, ~8 min) during this lesson. It explains the rationale for the classroom structure directly to students in a way that lands well.

Some teachers also use this lesson to co-create a class mission statement or list of success criteria with students. In practice, we found the tasks themselves set the tone without needing these additional activities. Budget time for them if you find them valuable, but do not feel obligated. The group work rubric is the essential culture artifact from this lesson.

Students make notes on:

- Organizing: what it means and how it reveals pattern (palindrome +110 example)
- Generalizing: introducing variables to move from specific cases to ‘always true’
- Group work rubric: students copy the final version into their notes

Feeds into

- Lesson 0.3: convincing arguments build directly on organizing and generalizing
- Unit 2: the algebraic generalization move (variables = ‘for every’) returns in the four forms of the conditional statement