

Check Your Understanding 1.4

Show all work. Use correct notation throughout.

Basic	Intermediate	Advanced
Row 2 Spiral — The Foundational Objects → Naming what we reason about		
<i>Basic — Spiral</i> 1. Write the correct notation for each. (a) <i>The line through J and K.</i> (b) <i>The ray starting at T through U.</i> (c) <i>The segment from W to V.</i> 2. Define collinear and coplanar in your own words.	<i>Intermediate — Spiral</i> 3. In a diagram, points A, B, and C are collinear. Point D is not on line AC. Name all the rays, segments, and lines you can form using these four points.	<i>Advanced — Spiral</i> 4. Explain carefully why point, line, and plane are undefined terms. Connect this to the classification and differentiation framework from Lesson 1.1: why does that framework make it impossible to define point and line?
Row 3 Spiral — Distance and Midpoint in the Plane → Measuring what we reason about		
<i>Basic — Spiral</i> 5. Find the distance and midpoint for each pair. (a) <i>A(-2, 5) and B(4, -3).</i> (b) <i>C(0, -6) and D(-3, 2).</i>	<i>Intermediate — Spiral</i> 6. <i>Segment AB has midpoint M(1, 4). Point A is at (-3, 2). Find B and the length of the full segment.</i> 7. <i>Points P(1, 2), Q(3, 6), and R(x, 10) are collinear. Find x.</i>	<i>Advanced — Spiral</i> 8. <i>A triangle has vertices A(0,0), B(10,0), C(x, y). The midpoints of AB, BC, and AC are $M_1(5, 0)$, $M_2(8, 4)$, and $M_3(3, 4)$ respectively. Find C.</i>
Row 4 Spiral — Polygons and Area in the Plane → Applying measurement to figures		
<i>Basic — Spiral</i> 9. <i>A triangle has vertices P(0,0), Q(8,0), R(3,6). Find its area.</i> 10. <i>Classify this polygon: a closed, non-self-intersecting figure with 8 sides where all diagonals remain inside.</i>	<i>Intermediate — Spiral</i> 11. <i>A polygon has vertices A(0,0), B(5,0), C(5,4), D(3,6), E(0,4). Find the perimeter.</i>	<i>Advanced — Spiral</i> 12. <i>A triangle has vertices A(0,0), B(6,0), C(2,5). Find the area two ways: (1) $\frac{1}{2} \times \text{base} \times \text{height}$, and (2) bounding rectangle minus extra triangles. Confirm both agree.</i>

Basic	Intermediate	Advanced
Angle Relationships → The first deductive results		
<i>Basic</i> 13. Find the complement and supplement of each angle. (a) 34° (b) 78° 14. $\angle 1$ and $\angle 2$ are a linear pair. $m\angle 1 = (3x + 10)^\circ$ and $m\angle 2 = (5x - 2)^\circ$. Find x and both angles. 15. Identify all pairs of vertical angles formed when two lines cross at point E.	<i>Intermediate</i> 16. Two lines intersect. One angle is $(4x + 15)^\circ$. Its vertical angle is $(6x - 5)^\circ$. Find x and all four angles. 17. $\angle YOZ$ and $\angle XOY$ are adjacent. $m\angle YOZ = 25^\circ$ and $m\angle XOZ = 85^\circ$. Find $m\angle XOY$. 18. Can two adjacent angles also be supplementary? Can they also be vertical? Explain each.	<i>Advanced</i> 19. Write a complete argument that vertical angles must always be congruent. State every assumption and every conclusion. Use the linear pair relationship as your starting point.
Algebra Spiral Exponent Properties — Quotient Rule and Negative Exponents	<i>Algebra Spiral</i> Simplify each expression. Write your answer without negative exponents. (a) $x^8 \div x^3$ (b) $(2x^3y^2) \div (4x^5y)$ (c) $3x^{-2} \cdot x^4$	

Answers — CYU 1.4**Row 2 Spiral**

- 1a. Line JK (double arrow over JK)
 1b. Ray TU (arrow from T through U)
 1c. Segment WV (bar over WV)
 2. Collinear: points that all lie on the same line. Coplanar: points or lines that all lie in the same plane.
 3. Rays: from each of A, B, C, D toward every other point. Segments: AB, AC, BC, AD, BD, CD. Lines: line AC (= AB = BC), line AD, line BD, line CD.
 4. A good definition classifies (names a larger group) and differentiates. There is no larger geometric category to classify point or line into. Any attempt to define them uses equally basic terms or goes in a circle. They are the irreducible foundation.

Row 3 Spiral

- 5a. Distance = 10. Midpoint = (1, 1).
 5b. Distance = $\sqrt{73}$. Midpoint = $(-3/2, -2)$.
 6. B = (5, 6). Length = $4\sqrt{5}$.
 7. Slope PQ = 2. $(10-2)/(x-1) = 2 \rightarrow x = 5$.
 8. M_3 midpoint of AC: x = 6, y = 8. C = (6, 8). Verify M_2 : midpoint BC = (8, 4). ✓

Row 4 Spiral

9. Area = $\frac{1}{2} \times 8 \times 6 = 24$.
 10. Convex octagon.
 11. AB = 5, BC = 4, CD = $2\sqrt{2}$, DE = $\sqrt{13}$, EA = 4. Perimeter ≈ 19.44 .
 12. Method 1: base = 6, height = 5. Area = 15. Method 2: bounding rect 30, minus left Δ (5) and right Δ (10). Area = 15. ✓

Angle Relationships

- 13a. Complement = 56° , supplement = 146° .
 13b. Complement = 12° , supplement = 102° .
 14. $8x + 8 = 180 \rightarrow x = 21.5$. $\angle 1 = 74.5^\circ$, $\angle 2 = 105.5^\circ$.
 15. $\angle 1$ and $\angle 3$ are vertical. $\angle 2$ and $\angle 4$ are vertical.
 16. $4x + 15 = 6x - 5 \rightarrow x = 10$. One angle = 55° . Vertical = 55° . Linear pair = 125° each.
 17. 60° .
 18. Adjacent and supplementary: yes — a linear pair is both. Adjacent and vertical: no — vertical angles are formed by opposite rays and cannot share a side.
 19. $\angle 1$ and $\angle 2$ form a linear pair $\rightarrow \angle 1 + \angle 2 = 180^\circ$. $\angle 2$ and $\angle 3$ form a linear pair $\rightarrow \angle 2 + \angle 3 = 180^\circ$. Both equal 180° , so $\angle 1 + \angle 2 = \angle 2 + \angle 3$. Subtract $\angle 2$: $\angle 1 = \angle 3$. Vertical angles are congruent. ■

Algebra Spiral

- (a) x^5
 (b) $y / (2x^2)$
 (c) $3x^2$