Lesson 8.1 — Teacher Reference Notes

What Is a Circle?

Unit 8: Circles • Day 1 of 7

At a Glance	
	Pacing: Open thinking task (~30–35 min) • Gallery walk (~5 min) • Making meaningful notes (~10–12 min) • Check Your Understanding (~15–20 min) **Materials:** Vertical whiteboards or large paper • String or rope (20–30 cm per group) • Rulers and markers • Coordinate grid drawn or projected for the equation phase **Nav instrument row active:** Row 1 (What is a circle? → Defining the object) activates today. This row remains active for the rest of the unit — every subsequent lesson rests on it. **Key setup note:** Distribute string before giving the task — but do not explain why. Watching groups figure out what the string is for is part of the lesson.

OBJECTIVES

By the end of this lesson students can:

- State a precise mathematical definition of a circle as the set of all points at a fixed distance from a center.
- Derive the equation of a circle $(x-h)^2 + (y-k)^2 = r^2$ from the definition and the distance formula.
- Given a circle's equation, identify the center and radius — including handling sign traps in $(x \pm h)$ and $(y \pm k)$.
- Determine whether a given point lies on, inside, or outside a circle, and explain why the method works.
- Find the equation of a circle given a center and a point on it, using the distance formula to compute the radius.

Narrative Arc — Open class with this — before revealing any task

"We are starting a new unit today: Circles. You have known what a circle is your whole life. But today I am going to ask you to define one — precisely, mathematically. And then I am going to ask you to write its equation, just from that definition. The whole rest of this unit will rest on what we do in the next ninety minutes."

TASK

Thinking Task — What Is a Circle? (≈30–35 min)

Give each group a length of string and a marker. Do not explain why yet. Give the task verbally:

"Using only a marker and a piece of string, draw every point on your board that is exactly the length of your string away from a fixed point of your choosing. Then, as a group, write a precise mathematical definition of the shape you drew."

That is the whole prompt. Do not add anything yet. Let groups work. As soon as a group has a precise definition, push that group forward immediately — do not pause for class discussion:

"Good. Now put that circle on a coordinate plane with center (h, k) and radius r. A point (x, y) is on the circle if and only if... what? Can you write an equation for that?"

Three nudges if a group is stuck on the equation:

- “What is the distance from (x, y) to (h, k)? You have a formula for that.”
- “If the distance equals r, what equation does that give you?”
- “Why might you want to square both sides?”

Common student definitions and how to push

“A round shape” → “Would an oval satisfy your definition? What makes this different from an oval?”

“All points the same distance from the middle” → “Good — distance from what exactly? The same for all of them, or just most?”

“The set of all points exactly r from a given point” → “Excellent. How many points are in that set? What does ‘set of all points’ really mean?”

The phrase ‘set of all points’ is genuinely strange when you think about it carefully, there are infinitely many such points, and together they form the circle. Students who reach this level of precision are ready to move to the equation.

Extension Ladder — For Groups That Keep Going

Give these verbally as groups are ready, one at a time. Do not hand them out all at once.

Level 1: Write the equation of the circle with center (3, −2) and radius 5. Then one with center (0, 0) and radius 1. What is special about the second circle?

Level 2: Given $(x + 1)^2 + (y - 4)^2 = 49$, identify center and radius. Does (6, 4) lie on, inside, or outside the circle? Justify.

Level 3: A circle has center (3, 4) and passes through the origin. Find its equation. What tool did you need to find the radius?

Level 4: Two circles: center (0, 0) with radius 5; center (4, 0) with radius 3. How far apart are the centers? Do the circles overlap, touch, or not meet at all? Justify.

Level 5 — Hard: Three points all lie on the same circle: (1, 0), (−1, 0), and (0, 1). Part A: find the equation. Part B: now try (0, 0), (4, 0), and (0, 4). This requires solving a system of three equations. You have only solved systems of two before — can you extend the method?

⚠ Protect the derivation

The equation of a circle is often presented as a formula to memorize. Here it must emerge from two ideas students already own — the definition of a circle and the distance formula — by setting the distance equal to r and squaring. Students who derive it own it. Students who are shown it tend to confuse it with other formulas (especially the line-distance formula and the slope formula). If a group jumps to the answer because they remember it from middle school, ask them: “Why does that formula work? Where does it come from?” The point of the lesson is the derivation, not the formula itself.

CONSOLIDATION — WHAT TO SURFACE

As groups write notes, circulate and mark two or three boards to share. Surface these in this order:

What to surface (in order)

Best definition: The set of all points in a plane at a fixed distance r from a fixed center. Emphasize 'set of all points' — the circle is the boundary only, not the filled region.

The derivation: Distance formula applied to the definition. Setting $\sqrt{(x-h)^2+(y-k)^2} = r$, then squaring. This is just the Pythagorean theorem in coordinate form.

Connection to prior work: The distance formula is from Unit 1. The Pythagorean theorem is from even earlier. The equation of a circle is not new mathematics — it is a new application of old tools.

The unit circle: When center = (0,0) and r = 1, the equation becomes $x^2+y^2=1$. Worth naming explicitly — students will see it again in precalculus.

Sign traps: $(x+1)^2+(y-4)^2=49$ has center (−1, 4), not (1, −4). The center coordinates have opposite signs to what they appear in the equation. Address this directly — it is the most common error on the CYU.

The Equation — for the Board

A circle with center (h, k) and radius r has equation:

$$(x - h)^2 + (y - k)^2 = r^2$$

Why: A point (x, y) is on the circle if and only if its distance from (h, k) equals r. The distance is $\sqrt{(x-h)^2+(y-k)^2}$. Setting that equal to r and squaring gives the equation.

▶ Narrative Arc — End of class — transition to Lesson 8.2

"You now have the definition of a circle and its equation. Tomorrow we are going to ask a question that sounds easy but turns out to be remarkable: are all circles similar? Can a tiny circle and an enormous circle be the same shape? You already know the answer, intuitively. Tomorrow we are going to prove it and we will see something about circles that is true of almost no other shape in geometry."

✎ Students make notes on:

- A precise mathematical definition of a circle — 'the set of all points in a plane at a fixed distance r from a fixed center.'
- The equation of a circle $(x - h)^2 + (y - k)^2 = r^2$, with a short derivation in their own words showing where it comes from.
- What other equation/theorem the equation of a circle is the same as, or closely related to (Pythagorean theorem via the distance formula).
- Common mistakes to remember — especially the sign trap with $(x \pm h)$ and $(y \pm k)$.
- Three worked examples: (a) given center and radius, write the equation; (b) given the equation, find center and radius; (c) given a point, determine if it is on, inside, or outside the circle, and why the method works.

Connection Points	**Draws from Unit 1:** The distance formula is the core tool. It is just the Pythagorean theorem written for two points in the coordinate plane. **Sets up Lesson 8.2:** The one-parameter idea — a circle is determined by a single number, its radius — is the engine of tomorrow's lesson on why all circles are similar. **Sets up the whole unit:** Every major theorem in Unit 8 — inscribed angle, chord-bisector, radius-tangent, Law of Sines as a circle theorem — ultimately traces back to the single fact established today: all points on a circle are the same distance from the center.